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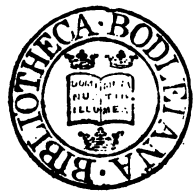

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INTRODUCTION.

No branch of the Science of Mathematics is of more extensive application, and greater practical usefulness, than that which is embodied under the term "Mensuration." For the measurement of Distance, Space, and Solidity, is one of the first things upon which the judgment of every child is exercised; and questions of more or less importance, depending for their solution upon a knowledge of at least the first principles of this science, are of every day occurrence. Such, for example, is the quantity of carpet required for the floor, or of paper for the walls, of a room; of lead for a pipe or cistern; glass for a conservatory; turf for a lawn; trees for a shrubbery, &c. &c. In the great majority of these and the like cases, employers are completely at the mercy of their tradesmen; who again, on the other hand, from imperfect knowledge of Mensuration, often make wrong estimates, and in consequence enter into engagements which they cannot fulfil without serious loss to themselves.

It seems singular, therefore, that the study of this subject should have been so much neglected hitherto in schools, and that ignorance of it should still so exten-

sively prevail. This is more to be wondered at when it is considered how dry and uninteresting Euclid usually appears to students, from lack of practical illustration; and that Trigonometry and Conic Sections are rendered needlessly difficult to most beginners, by their having had little or no previous acquaintance with those important elementary truths which form a necessary introduction to the satisfactory study of either.

There is, however, good reason for believing that the neglect in question is attributable, not so much to an under-estimate of the value of Mensuration, as to an over-estimate of its difficulty; and that this has arisen mainly from the intricate and voluminous character of the various works treating of it, which has rendered them quite unsuitable for ordinary use in schools.

To do something towards obviating this defect, and supplying a remedy, is the object of the present little work. It does not in the slightest degree aim at superseding any of the existing treatises on the subject, but rather proposes to furnish a convenient stepping-stone to them, by the aid of which students may become thoroughly familiar with the elementary processes of Geometry, and so find less difficulty in advancing along the thorny paths of the higher mathematics.

For the solution of all but a very few of the questions here proposed, nothing is requisite beyond a knowledge of Vulgar and Decimal Fractions, Square Root, and Simple Equations, so that the book may be advantageously placed in the hands even of junior students. The examples will be found also to afford excellent practice to more advanced pupils in all the ordinary operations of arithmetic; thus tending to promote

general accuracy and exactness, while illustrating a subject at once novel and interesting.

It has been the Author's special study to attain to clearness and brevity in all explanations, and to reduce everything to *first principles*, so as to render every new fact throughout the book easy, not only to be comprehended, but retained, even by the *self-instructor*. With this view, he has avoided mere formulæ and barren rules as much as possible, adopting very few indeed, and then only from necessity, and labouring to impress these upon the memory by recurrence to them, and deduction from them wherever the subject would admit. With the same object, the headings or "Cases" have been varied, rather than examples multiplied under any single one, so as still to produce a sufficient number upon the whole, without causing any one portion to seem wearisome to the learner.

The Square, Oblong, and Triangle, have been worked out more at length, and illustrated more copiously than any other figures, on account of their greater practical importance as occurring more frequently than the rest; being, in fact, the foundation of them all, and therefore well worth attentive study.

On the subject of "Duodecimals" or Multiplication by Feet, Inches, Parts, &c., there will of course be different opinions; but the Author has here taken the liberty of studiously ignoring the whole system, on the ground of its being not only quite needless in itself (at least to 999 out of every 1000 students), but, like Troy and Apothecaries' Weight among the "Tables," productive of much positive mischief, by creating

a vast confusion of ideas, and unnecessarily complicating a subject already sufficiently difficult to beginners.

The Appendix contains much important and interesting matter, not ordinarily attainable except by reference to a variety of expensive and intricate volumes. And the whole work, having been drawn up by a practical schoolmaster for the use of his own pupils, (who have in fact done every question here presented,) will, it is believed, be found sufficiently comprehensive and comprehensible to serve as a convenient manual or text-book on the subject for *Masters of National Schools, Mechanics' Institutes*, and *Working Men's Colleges*, as well as for all who are engaged in the arduous task of preparation for appointments in the Civil, Military, or Naval Services.

It remains only to say, that, notwithstanding all the care and attention bestowed upon the work, errors will doubtless have here and there crept in, and that the Author will feel greatly obliged to any one for *well-considered* corrections, as well as for hints and suggestions which may be available for the improvement of another edition.

J. S. B.

Feb. 2. 1857.

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[*N. B.*—The student need not wait till he has gone through the whole of Plane Mensuration before beginning Solids, but may commence as soon as he has finished the Square and Oblong, and let the two branches of the subject run parallel with each other throughout.]

THE FOLLOWING ABBREVIATIONS ARE USED OCCASIONALLY.

=	for	equals.	⊙	for	circle.
∴	„	therefore.	⊙ce	„	circumference.
∠	„	angle.	cr.	„	centre.
rt. ∠	„	right angle.	diam.	„	diameter.
⊥	„	perpendicular.	rad.	„	radius.
∥	„	parallel.	diagl.	„	diagonal.
⌑	„	parallelogram.	6gon or 6n	&c.	Hexagon, &c.
△	„	triangle.			

MENSURATION.

PART I.

MEASUREMENT OF PLANE FIGURES, OR FLAT SURFACES.

1. SUPERFICIES or surface has only two dimensions, viz. length and breadth.

2. The superficial or surface measurement of any space or figure is called its *area*.

3. When the figure is rectangular, *i. e.* when its sides are four straight lines at right angles or perpendicular to each other, (*e. g.* a square, or oblong), its area is found by ***multiplying together its length and breadth.***

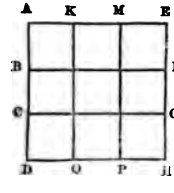
4. Any two of these being given, we can find the third:

$$\left.\begin{array}{ll} \text{For since} & \text{Length} \times \text{Breadth} = \text{Area}, \\ \text{It follows, that} & \text{Length} = \dfrac{\text{Area}}{\text{Breadth}} \\ \text{and} & \text{Breadth} = \dfrac{\text{Area}}{\text{Length}} \end{array}\right\}$$

5. To determine the number of square feet in a square yard.

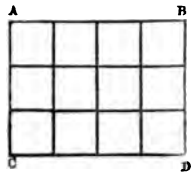
Take any square surface measuring a yard along each side (*Fig.* 1.) On either pair of opposite sides place marks, as A, B, C, D, and E, F, G, H, exactly one foot apart; and draw straight lines joining B F and C G. The square is then divided into 3 equal spaces, each 3 feet long and 1 foot broad. Now place marks, as K, M, O, P, on the other pair of sides, and join them, as before. Each of the 3 spaces will then be divided into

Fig. 1.



3 equal figures, each 1 foot long and 1 foot broad; *i. e.* into 3 square feet: and the total number of these in the square yard will therefore be 9. Similarly, a square foot may be lined out into 144 square inches: — and so on.

Fig. 2.



6. In the same way may be calculated the area of an oblong (*Fig.* 2.) Hence the general rule for determining the area of any rectangular surface, viz., *find the product of length* × *breadth.*

7. Take the case of a window, 3 panes in width and 4 in height. Then the total number of panes may be reckoned, by counting them as either 3 rows with 4 in each, or 4 rows with 3 in each, *i. e.* 3×4, or 4×3; each way making 12 altogether.

8. This consideration renders easy the computation of numbers in all oblong figures where the units are massed together regularly, *e. g.* soldiers in a column, trees in a plantation, bricks or stones in a wall, &c. And even where irregularity prevails, as in crowds of people, &c. an estimate may be made with tolerable accuracy by the same method.

9. Resuming the illustration in note 7, of a window 3 panes wide and 4 high. Suppose the dimensions of each pane to be 11 inches wide, and 18 high; then the total area of glass in the window may be computed thus: —

Area of 1 pane = 18 × 11 inches = 198 sq. inches.
∴ do. 12 = 198 × 12 = 2376 do.

Or thus: —

Total width of window = 11 × 3 = 33 inches;
Total height do. = 18 × 4 = 72 do;
∴ Total area do. = 72 × 33 = 2376 sq. inches.

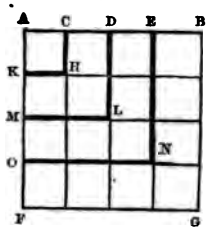
the result being the same either way.

10. Observe the difference between "feet square" and "square feet." To say that any space measures "3 feet square," implies that *each side* of it is 3 feet long, and consequently that it contains 9 *square feet;* while the words "3

square feet" mean only that a figure of that extent contains as much space as 3 squares, each 1 foot long and 1 foot broad. "Three feet square" therefore, is just three times the size of "three square feet."

11. *Fig.* 3. will illustrate what is meant when it is said that "the square of $\frac{1}{2} = \frac{1}{4}$;" that the "square root of $\frac{1}{16} = \frac{1}{4}$," &c. For A D is $\frac{1}{2}$ of A B; and A D L M, which is the square of A D, is exactly $\frac{1}{4}$ of A B F G, the square of A B. Again, A C is $\frac{1}{4}$ of A B, and A C H K, its square, is $\frac{1}{16}$ of A B F G; so that the square root of A C H K, *i. e.* the *length of one side*, as A C, is $\frac{1}{4}$ the length of A B, a side of that square A B F G, which is 16 times the size of A C H K.

Fig. 3.



12. Similarly A E is $\frac{3}{4}$ of A B; and A E N O, its square, is $\frac{9}{16}$ of A B F G, *i. e.* contains 9 out of the 16 equal portions which make up the whole figure. Therefore, that $(\frac{3}{4})^2 = \frac{9}{16}$ and $\sqrt{\frac{9}{16}} = \frac{3}{4}$, is true of *space* as well as *number*. And so with other quantities in the same way.

13. Since area = length × breadth, it follows that if in any case *one* only of these dimensions be altered, then the area will be altered in *simple* proportion to this change; but if *both* be altered, the area is altered in *compound* proportion to both.

e. g. A garden 50 yds. long, and 30 wide, or (50 × 30) sq. yds, has its *length* doubled. Then its area becomes {(50 × 2) × 30} sq. yds., or just *twice* its former size: while if its *breadth* be doubled too, its area becomes {(50 × 2) × (30 × 2)} or {(50 × 30) × 4} sq. yds., *i. e.* just *four* times its size originally.

14. In the same way, if *both* dimensions be trebled, quadrupled, quintupled, &c., then the area becomes 9, 16, 25 &c. times what it was at first; while if one dimension be trebled and the other doubled, area = (3 × 2) or 6 times the original.

And so of any other increase or diminution in the length of the sides.

15. If one dimension be *doubled*, &c., while the other is *halved*, &c., no change at all takes place in the area. And with these principles in view, several questions may be readily formed, for further practice, out of each one that is here given; the answers being known instantly from the one already discovered, without the labour of working each question over again from the beginning.

16. On the same principle must alterations be made *proportionally* in the length and breadth of any space, in order to effect any wished-for *proportional* increase or diminution in its area.

e. g. Suppose it requisite to *double* the size of the garden in the last example, without altering the *proportion* of its sides, *i. e.* still keeping them in the proportion of 50 to 30, or 5 : 3.

Let A represent area; L and B, length and breadth.

Then, since if 4 A be wanted, we must have

			2 L × 2 B, *i. e.* $\sqrt{4}$ L × $\sqrt{4}$ B.
„	if 9 A	„	3 L × 3 B, *i. e.* $\sqrt{9}$ L × $\sqrt{9}$ B.
„	if $\frac{1}{4}$ A	„	$\frac{1}{2}$ L × $\frac{1}{2}$ B, *i. e.* $\sqrt{\frac{1}{4}}$ L × $\sqrt{\frac{1}{4}}$ B.
	&c.		&c.

It follows by the same law, that

if 2 A	be wanted,	we must have	$\sqrt{2}$ L × $\sqrt{2}$ B,
if 3 A	„	„	$\sqrt{3}$ L × $\sqrt{3}$ B,
if $\frac{1}{2}$ A	„	„	$\sqrt{\frac{1}{2}}$ L × $\sqrt{\frac{1}{2}}$ B.
&c.			&c.

We can now determine the new dimensions of the garden.

For since 2 A is wanted, the sides will be $\sqrt{2}$ times 50, and $\sqrt{2}$ times 30, *i. e.* 70·71 yds. and 42·426 yds.; which numbers are exactly in the proportion of 5 : 3, as required, while their *product* gives the requisite double area as nearly *as the use of* decimals renders possible.

MEASURES OF DISTANCE AND SPACE.

NOTE. A *linear* yard means a yard *long*; being merely the measure of a *line*, which has *length*, but neither breadth nor thickness.

Distance.	*Space.*
12 linear inches = 1 linear foot.	144 square inches (*i.e.* 12 × 12) = 1 square foot.
3 linear feet = 1 linear yard.	9 square feet (*i.e.* 3 × 3) = 1 square yard.
5½ linear yards = 1 linear pole, or rod, or perch.	30¼ square yds. (*i.e.* 5½ × 5½) = 1 square pole, &c.
1760 linear yards = 1 linear mile.	40 square poles (*i.e.* 1210 square yds.) = 1 rood.
	4 roods (*i.e.* 160 p. or 4840 yds.) = 1 acre.

Land Measure by Gunter's Chain.

100 linear links	= 1 linear chain (4 poles or 22 yds. long.)
10,000 square links	= 1 sq. chain (484 sq. yds.)
10 sq. chns. (100,000 lks.)	= 1 acre. (4840 sq. yds.)

It will be seen from the above, that the length of the chain has been fixed at 22 yds., merely for the purpose of facilitating calculation: the process of conversion from links and chains into yards, acres, &c., or *vice versâ*, being peculiarly easy in consequence; while its division into 100 links answers the same purpose, since links may be considered as decimal parts of a chain, and both square links and square chains as decimal parts of an acre. Thus, 423 links are the same as 4 ch. 23 lks., or 4·23 chains. And the area of a square, of which this is a side, may be written either as 178,929 sq. lks., 17·8929 sq. ch., 1·78929 acres, or 1 acre, 7 ch., 8929 lks.

To convert square links and chains into acres, roods, poles, &c., and *vice versâ*.

ch. lks.

Ex. 1. 17 9750 = 1·79750 acres. Now by multiplying the decimal part of an acre by 4, 40, $30\frac{1}{4}$, &c. in succession, we get roods, poles, yards, &c.

Thus,—

	acre.
	·79750
	4
roods	3·19000
	40
poles	7·60000
	$30\frac{1}{4}$
	1800000
	15000
yds.	18·15000

ch. lk. a. r. p. y.
∴ 17 9750 = 1 3 7 $18\frac{3}{20}$.

r. p. a. r. p. r. p. p.
Ex. 2. 22 23 = 5 2 23. Now 2 23 = 103 = $\frac{103}{160}$ of an acre, (since 160 p. = 1 acre.)

lks. lks.
= $\frac{103}{160}$ of 100,000 lks. = 103 × 625 = 64375.

r. p. acres. a. ch. lks.
∴ 22 23 = 5·64375 = 5 6 4375.

Land Measure.

I. Transform into acres, roods, poles, and yards; —

15 c. 5000 lks.; 13 c. 5625 lks.; 3 c. 8750 lks.; 40 c. 6875 lks.; 2 c. 7125 lks.; 18 c. 9875 lks.

II. Transform into acres, chains, and links; —

18 r. 24 p.; 27 r. 5 p.; 17 r. 39 p.; 31 r. 5 p.; 9 r. $29\frac{3}{5}$ p.; 2 r. 6 p. $24\frac{1}{5}$ yds.

Answers.

I. 1 a. 2 r. 8 p.; 1 a. 1 r. 17 p.; 1 r. 22 p.; 4 a. 0 r. 11 p.; 1 r. 3 p. $12\frac{1}{10}$ yds.; 1 a. 3 r. 23 p. $24\frac{1}{5}$ yds.

II. 4 a. 6 c. 5000 lks.; 6 a. 7 c. 8125 lks.; 4 a. 4 c. 9375 lks.; 7 a. 7 c. 8125 lks.; 2 a. 4 c. 3500 lks.; 5 c. 4250 lks.

THE SQUARE.

Case I. [N.B. Area of any square = the square of one side.] *Vid. Figs.* 1. and 3.

I. Find the area in sq. yds. of each of the following square figures, the sides of which respectively are: —

19 yds.; 38 yds.; 87 yds.; 174 yds.; 825 yds.; 275 yds.; 178 yds.; 534 yds.; 97 yds.; 388 yds.

II. Do. do. [*Vid.* "Carpet and Paper," *infrà*, note to Question 5.]

$19\frac{3}{4}$ yds.; $39\frac{1}{2}$ yds.; $9\frac{7}{8}$ yds.; $27\frac{2}{3}$ yds.; $13\frac{5}{6}$ yds.; $16\frac{11}{12}$ yds.; $9\frac{3}{7}$ yds.; $5\frac{4}{9}$ yds.; $47\frac{4}{5}$ yds.; $11\frac{19}{20}$ yds.

[The student may work the above both *with* and *without* decimals, making the one method prove the other.]

III. Find area, in yds., as before; —

84 ft.; 56 ft.; 28 ft.; 346 ft.; 519 ft.; 173 ft.; 645 ft.; 215 ft.; 93 ft.; 372 ft.

IV. Do. do.

$83\frac{7}{8}$ ft.; $27\frac{2}{3}$ ft.; $39\frac{1}{2}$ ft.; $41\frac{15}{16}$ ft.; $13\frac{5}{6}$ ft.; $19\frac{3}{4}$ ft.

V. Do. do.

97 in.; 291 in.; 194 in.; 789 in.; 263 in.; 526 in.; 68 in.; 272 in.; 204 in.; 136 in.

VI. Do. do.

$195\frac{1}{2}$ in.; $48\frac{7}{8}$ in.; $89\frac{9}{10}$ in.; $97\frac{3}{4}$ in.; $87\frac{2}{3}$ in.; $179\frac{4}{5}$ in.

VII. Find area, in yds., ft., and in.: —

79 ft. 6 in.; 27 ft. 10 in.; 39 ft. 9 in.; 55 ft. 8 in.; 49 ft. 3 in.; 112 ft. 8 in.; 34 ft. 7 in.; 98 ft. 6 in.; 56 ft. 4 in.; 69 ft. 2 in.

VIII. Find area, in acres, roods, and poles, of &c.

57 p.; 43 p.; 38 p.; 29 p.; 47 p.; 54 p.

IX. Do. do.

$57\frac{1}{2}$ p.; $43\frac{1}{4}$ p.; $38\frac{3}{4}$ p.; $29\frac{1}{2}$ p.; $47\frac{1}{4}$ p.; $54\frac{3}{4}$ p.

X. Find area, in acres, roods, poles, and yards, of &c. :—

89 yds. ; 356 yds. ; 267 yds. ; 178 yds. ; 548 yds. ; 411 yds. ; 137 yds. ; 274 yds.

XI. Find area, in acres, chains, and links, of &c. : —

141 lks. ; 133 lks. ; 705 lks. ; 423 lks. ; 399 lks. ; 564 lks. ; 266 lks. ; 846 lks. ; 987 lks. ; 798 lks. ; 665 lks. ; 1974 lks. ;

XII. 8 ch. 63 lks. ; 6 ch. 27 lks. ; 2 ch. 66 lks. ; 9 ch. 87 lks. ; 17 ch. 26 lks. ; 18 ch. 81 lks. ; 7 ch. 98 lks. ; 25 ch. 08 lks. ; 8 ch. 46 lks. ; 34 ch. 52 lks. ; 25 ch. 89 lks. ; 19 ch. 74 lks.

Answers to Case I.

I. 361 ; 1444 ; 7569 ; 30276; 680625 ; 75625 ; 31684 ; 285156 ; 9409 ; 150544 ; sq. yds.

II. $390\frac{1}{16}$; $1560\frac{1}{4}$; $97\frac{33}{64}$; $765\frac{4}{9}$; $191\frac{13}{36}$; $286\frac{25}{144}$; $88\frac{44}{49}$; $29\frac{52}{81}$; $2284\frac{21}{25}$; $142\frac{221}{400}$; (all sq. yds.)

III. 784 ; $348\frac{4}{9}$; $87\frac{1}{9}$; $13301\frac{7}{9}$; 29929 ; $3325\frac{4}{9}$; 46225 ; $5136\frac{1}{9}$; 961 ; 15376 ; (all sq. yds.)

IV. 781 yds. $6\frac{1}{64}$ ft. ; $85\frac{4}{81}$ yds. ; 173 yds. $3\frac{1}{4}$ ft. ; 195 yds. $3\frac{103}{256}$ ft. ; 21 yds. $2\frac{13}{36}$ ft. ; 43 yds. $3\frac{1}{16}$ ft.

V. 7 yds. 2 ft. 49 in. ; 65 yds. 3 ft. 9 in. ; 29 yds. 0 ft. 52 in. ; 480 yds. 3 ft. 9 in. ; 53 yds. 3 ft. 49 in. ; 213 yds. 4 ft. 52 in. ; 3 yds. 5 ft. 16 in. ; 57 yds. 0 ft. 112 in. ; 32 yds. 1 ft. ; 14 yds. 2 ft. 64 in.

VI. 29 yds. 4 ft. $60\frac{1}{4}$ in. ; 1 yd. 7 ft. $84\frac{49}{64}$; 6 yds. 2 ft. $18\frac{1}{100}$ in. ; 7 yds. 3 ft. $51\frac{1}{16}$ in. ; 5 yds. 8 ft. $53\frac{4}{9}$ in. ; 24 yds. 8 ft. $72\frac{1}{25}$ in.

VII. 702 yds. 2 ft. 36 in. ; 86 yds. 0 ft. 100 in. ; 175 yds. 5 ft. 9 in. ; 344 yds. 2 ft. 112 in. ; 269 yds. 4 ft. 81 in. ; 1410 yds. 3 ft. 112 in. ; 132 yds. 8 ft. 1 in. ; 1078 yds. 0 ft. 36 in. ; 352 yds. 5 ft. 64 in. ; 531 yds. 5 ft. 4 in.

VIII. 20 a. 1 r. 9 p. ; 11 a. 2 r. 9 p. ; 9 a. 0 r. 4 p. ; 5 a. 1 r. 1 p. ; 13 a. 3 r. 9 p. ; 18 a. 0 r. 36 p.

IX. 20 a. 2 r. $26\frac{1}{4}$ p. ; 11 a. 2 r. $30\frac{9}{16}$ p. ; 9 a. 1 r. $21\frac{9}{16}$ p. ; 5 a. 1 r. $30\frac{1}{4}$ p. ; 13 a. 3 r. $32\frac{9}{16}$ p. ; 18 a. 2 r. $37\frac{9}{16}$ p.

X. 1 a. 2 r. 21 p. $25\frac{3}{4}$ yds.; 26 a. 0 r. 29 p. $18\frac{3}{4}$ yds.; 14 a. 2 r. 36 p. 20 yds.; 6 a. 2 r. 7 p. $12\frac{1}{4}$ yds.; 62 a. 0 r. 7 p. $12\frac{1}{4}$ yds.; 34 a. 3 r. 24 p. 5 yds.; 3 a. 3 r. 20 p. 14 yds.; 15 a. 2 r. 1 p. $25\frac{3}{4}$ yds.

XI. 1 ch. 9881 lks.; 1 c. 7689 lks.; 4 a. 7 c. 7025 lks.; 1 a. 7 c. 8929 lks.; 1 a. 5 c. 9201 lks.; 3 a. 1 c. 8096 lks.; 7 c. 0756 lks.; 7 a. 1 c. 5716 lks.; 9 a. 7 c. 4169 lks.; 6 a. 3 c. 6804 lks.; 4 a. 4 c. 2225 lks.; 38 a. 9 c. 6676 lks.

XII. 7 a. 4 c. 4769 lks.; 3 a. 9 c. 3129 lks.; 7 c. 0756 lks.; 9 a. 7 c. 4169 lks.; 29 a. 7 c. 9076 lks.; 35 a. 3 c. 8161 lks.; 6 a. 3 c. 6804 lks.; 62 a. 9 c. 0064 lks.; 7 a. 1 c. 5716 lks.; 119 a. 1 c. 6304 lks.; 67 a. 0 c. 2921 lks.; 38 a. 9 c. 6676 lks.

Case II. To find the value of any square surface, the length of a side being given.

Ex. A square garden is 50 yds. long, and the land is valued at £60 10*s.* per acre. Find value of garden.

No. of sq. yds. in garden = 50 × 50 = 2500.

And at £60 10*s.* for 1 acre, or 4840 yds., 1 sq. yd. is worth 3*d.*

∴ Value of garden = 3*d.* × 2500 = £31 5*s.*

I. Find value, at 3*s.* per square foot, of the following squares, of which the sides are respectively, —

97 in.; 291 in.; 194 in.; 789 in.; 263 in.; 526 in.; 68 in.; 272 in.; 204 in.; 136 in.

II. Find value, at 6*s.* 9*d.* per sq. yd., of &c.

79 ft. 6 in.; 27 ft. 10 in.; 39 ft. 9 in.; 55 ft. 8 in.; 49 ft. 3 in.; 112 ft. 8 in.; 34 ft. 7 in.; 98 ft. 6 in.; 56 ft. 4 in.; 69 ft. 2 in.

III. Find value, at 5*d.* per sq. yd., of &c.

57 p.; 43 p.; 38 p.; 29 p.; 47 p.; 54 p.

IV. Find value, at 10*s.* 1*d.* per sq. pole, of, &c.

$54\frac{3}{4}$ p.; $47\frac{1}{4}$ p.; $29\frac{1}{2}$ p.; $38\frac{3}{4}$ p.; $43\frac{1}{4}$ p.; $57\frac{1}{2}$ p.

V. Find value, at £60 10*s.* per acre, of &c.

89 yds.; 356 yds.; 267 yds.; 178 yds.; 548 yds.; 411 yds.; 137 yds.; 274 yds.

VI. Find value, at $\frac{1}{4}d.$ per sq. link, of, &c.

141 lks.; 133 lks.; 705 lks.; 423 lks.; 399 lks.; 564 lks.

VII. Find value, at £5 4*s.* 2*d.* per sq. chain, of, &c.

2 ch. 66 lks.; 8 ch. 46 lks.; 9 ch. 87 lks.; 7 ch. 98 lks.; 6 ch. 65 lks.; 19 ch. 74 lks.

VIII. Find value, at £78 2*s.* 6*d.* per acre, of &c.

8 ch. 63 lks.; 6 ch. 27 lks.; 2 ch. 66 lks.; 9 ch. 87 lks.; 17 ch. 26 lks.; 18 ch. 81 lks.

Answers to Case II.

I. £9 16*s.* $0\frac{1}{4}d.$; £88 4*s.* $2\frac{1}{4}d.$; £39 4*s.* 1*d.*; £648 9*s.* $2\frac{1}{4}d.$; £72 1*s.* $0\frac{1}{4}d.$; £288 4*s.* 1*d.*; £4 16*s.* 4*d.*; £77 1*s.* 4*d.*; £43 7*s.*; £19 5*s.* 4*d.*

II. £237 0*s.* $2\frac{1}{4}d.$; £29 1*s.* $0\frac{1}{4}d.$; £59 5*s.* $0\frac{9}{16}d.$; £116 4*s.* 1*d.*; £90 19*s.* $2\frac{1}{16}d.$; £476 0*s.* 4*d.*; £44 17*s.* $0\frac{1}{16}d.$ £363 16*s.* $8\frac{1}{4}d.$; £119 0*s.* 1*d.*; £179 8*s.* $0\frac{1}{4}d.$

III. £2047 10*s.* $11\frac{1}{4}d.$; £1165 5*s.* $1\frac{1}{4}d.$; £910 0*s.* 5*d.*; £530 0*s.* $1\frac{1}{4}d.$; £1392 2*s.* $7\frac{1}{4}d.$; £1837 13*s.* 9*d.*

IV. £1511 5*s.* $5\frac{1}{16}d.$; £1125 11*s.* $8\frac{1}{16}d.$; £438 15*s.* $0\frac{1}{4}d.$; £757 0*s.* $9\frac{1}{16}d.$; £943 1*s.* $6\frac{1}{16}d.$; £1666 18*s.* $0\frac{1}{4}d.$

V. £99 0*s.* 3*d.*; £1584 4*s.*; £891 2*s.* 3*d.*; £396 1*s.*; £3753 16*s.*; £2111 10*s.* 3*d.*; £234 12*s.* 3*d.*; £938 9*s.*

VI. £20 14*s.* $2\frac{1}{4}d.$; £18 8*s.* $6\frac{1}{4}d.$; £496 18*s.* $0\frac{1}{4}d.$; £186 7*s.* $8\frac{1}{4}d.$; £165 16*s.* $8\frac{1}{4}d.$; 331 £7*s.*

VII. £36 17*s.* $0\frac{1}{2}d.$; £372 15*s.* $4\frac{1}{2}d.$; £507 7*s.* $7\frac{1}{8}d.$; £331 13*s.* $4\frac{1}{2}d.$; £230 6*s.* $6\frac{1}{8}d.$; £2029 10*s.* $4\frac{1}{2}d.$

VIII. £581 17*s.* $0\frac{3}{16}d.$; £307 2*s.* $7\frac{1}{16}d.$; £55 5*s.* $6\frac{3}{4}d.$; £761 1*s.* $4\frac{1}{16}d.$; £2327 8*s.* $0\frac{3}{4}d.$; £2764 3*s.* $9\frac{3}{16}d.$

Case III. Given, area of a square; find length of side.

N. B. Side of square $= \sqrt{\text{area}}$.

Ex. 1. Square contains 1225 sq. yds.

$\therefore$ Side $= \sqrt{1225} = 35$ yds.

Ex. 2. Square contains $\frac{49}{64}$ sq. yds.

$\therefore$ Side $= \sqrt{\frac{49}{64}} = \frac{7}{8}$ yd. $= 31\frac{1}{2}$ inches.

Ex. 3. Square contains $5\frac{1}{16}$ sq. yds.

$\therefore$ Side $= \sqrt{5\frac{1}{16}} = \sqrt{\frac{81}{16}} = \frac{9}{4} = 2\frac{1}{4}$ yds.

Obs. Ex. 2. and 3. might have been done by decimals; since $\frac{49}{64} = 765625$, and $5\frac{1}{16} = 5{\cdot}0625$.

I. Find length of one side in each of the following squares. Result in yds. ft. and in.

285126 yds.; 680625 yds.; 150544 yds.; 1560·25 yds.; 390·0625 yds.; 142·8025 yds.; 2284·84 yds.; 97·515625 yds.

II. Do. do.

$\frac{4}{9}$ yds.; $\frac{9}{16}$ yds.; $\frac{25}{36}$ yds.; $\frac{361}{1444}$ yds.; $\frac{31684}{28561}$ yds.; $\frac{2409}{10000}$ yds.; $2\frac{1}{4}$ yds.; $3\frac{1}{16}$ yds.; $4\frac{21}{25}$ yds.; $191\frac{3}{36}$ yds.; $88\frac{44}{49}$ yds.; $97\frac{33}{64}$ yds.; $286\frac{25}{144}$ yds.; $2284\frac{21}{25}$ yds.; $29\frac{52}{81}$ yds.

III. Do. do.

43 yds. $3\frac{1}{16}$ ft.; 781 yds. $6\frac{1}{4}$ ft.; 65 yds. 3 ft. 9 in.; 213 yds. 4 ft. 52 in.; 269 yds. 4 ft. 81 in.; 1410 yds. 3 ft. 112 in.; 29 yds. 4 ft. $60\frac{1}{4}$ in.; 24 yds. 8 ft. $72\frac{1}{25}$ in.

IV. Do. do. Result in yds.

20 a. 1 r. 9 p.; 13 a. 3 r. 9 p.; 11 a. 2 r. $30\frac{9}{16}$ p.; 18 a. 2 r. $37\frac{9}{16}$ p.; 34 a. 3 r. 24 p. 5 yds.; 6 a. 2 r. 7 p. $12\frac{1}{4}$ yds.

V. Do. do. Result in ch. and lks.

4 a. 9 c. 7025 lks.; 9 a. 7 c. 4169 lks.; 38 a. 9 c. 6676 lks.; 29 a. 7 c. 9076 lks.; 62 a. 9 c. 0064 lks.; 119 a. 1 c. 6304 lks.

Answers to Case III.

I. 534 yds.; 825 yds.; 388 yds.; 39 yds. 1 ft. 6 in.; 19 yds. 2 ft. 3 in.; 11 yds. 2 ft. $10\frac{1}{8}$ in.; 47 yds. 2 ft. $4\frac{4}{5}$ in.; 9 yds. 2 ft. $7\frac{1}{2}$ in.

II. 2 ft.; 2 ft. 3 in.; 2 ft. 6 in.; 1 ft. 6 in.; 12 in.; 9 in.; 1 yd. 1 ft. 6 in.; 1 yd. 2 ft. 3 in.; 2 yds. 0 ft. $7\frac{1}{8}$ in.; 13 yds. 2 ft. 6 in.; 9 yds. 1 ft. $3\frac{3}{7}$ in.; 9 yds. 2 ft. $7\frac{1}{2}$ in.; 16 yds. 2 ft. 9 in.; 47 yds. 2 ft. $4\frac{4}{5}$ in.; 5 yds. 1 ft. 4 in.

III. 6 yds. 1 ft. 9 in.; 27 yds. 2 ft. $10\frac{1}{4}$ in.; 8 yds. 0 ft. 3 in.;

14 yds. 1 ft. 10 in.; 16 yds. 1 ft. 3 in.; 37 yds. 1 ft. 8 in.; 5 yds. 1 ft. $3\frac{1}{2}$ in.; 4 yds. 2 ft. $11\frac{1}{2}$ in.

IV. $313\frac{1}{2}$ yds.; $258\frac{1}{2}$ yds.; $237\frac{1}{8}$ yds.; $301\frac{1}{8}$ yds.; 411 yds.; 178 yds.

V. 7 ch. 05 lks.; 9 ch. 87 lks.; 19 ch. 74 lks.; 17 ch. 26 lks.; 25 ch. 08 lks.; 34 ch. 52 lks.

Case IV. Given, value of a square; find length of side.

Ex. Value of a square garden is £31 5*s.* At £60 10*s.* per acre, find length of one side.

Now £60 10*s.* per acre is 3*d.* per sq. yard.

∴ There will be as many sq. yds. in the garden, as there are three-pences in its price.

$$\therefore \text{No. of sq. yds. in garden} = \frac{£31\ 5s.}{3d.} = \frac{7500d.}{3} = 2500.$$

$$\therefore \text{Length of one side} = \sqrt{2500} = 50 \text{ yds.}$$

I. Find length of one side in each of the following squares, the value of which, at 6*s.* 9*d.* per square yard, is respectively, —

£116 4*s.* 1*d.*; £363 16*s.* $8\frac{1}{4}d.$; £29 5*s.* $2\frac{1}{4}d.$; £90 19*s.* $2\frac{1}{16}d.$; £59 5*s.* $0\frac{9}{16}d.$; £44 17*s.* $0\frac{9}{16}d.$

II. Do. do. value at 7*s.* $6\frac{3}{4}d.$ per square pole.

£396 1*s.*; £938 9*s.*; £1584 4*s.*; £891 2*s.* 3*d.*; £2111 10*s.* 3*d.*; £3753 16*s.*

III. Do. do. value at £104 3*s.* 4*d.* per acre.

£331 7*s.*; £18 8*s.* $6\frac{1}{4}d.$; £20 14*s.* $2\frac{1}{4}d.$; £186 7*s.* $8\frac{1}{4}d.$; £165 16*s.* $8\frac{1}{4}d.$; £517 14*s.* $8\frac{1}{4}d.$

Answers to Case IV.

I. 18 yds. 1 ft. 8 in.; 32 yds. 2 ft. 6 in.; 9 yds. 0 ft. 10 in.; 16 yds. 1 ft. 3 in.; 13 yds. 0 ft. 9 in.; 11 yds. 1 ft. 7 in.

II. 178 yds.; 274 yds.; 356 yds.; 267 yds.; 411 yds.; 548 yds.

III. 5 ch. 64 lks.; 1 ch. 33 lks.; 1 ch. 41 lks.; 4 ch. 23 lks.; 3 ch. 99 lks.; 7 ch. 05 lks.

THE OBLONG.

Books, slates, envelopes, &c., are usually of this figure, which, like the square, is a rectangular parallelogram. It is often called simply a rectangle. (Euclid, bk. II.)

Case I. Given, length and breadth; find area. (*Fig.* 2.)

N.B. Area = length × breadth.

I. Find area of each of the following oblong figures, the dimensions of which are respectively,

13 and 11 yds.; 26 and 22 yds.; $19\frac{1}{2}$ and 17 yds.; $58\frac{1}{2}$ and 51 yds.; 19 and $17\frac{1}{2}$ yds.; $28\frac{1}{2}$ and $26\frac{1}{4}$ yds.; $9\frac{1}{3}$ and $8\frac{3}{4}$ yds.; $85\frac{1}{2}$ and $78\frac{3}{4}$ yds.

II. Do. do.

13 yds. 1 ft. and 11 yds. 2 ft.; 53 yds. 1 ft. and 46 yds. 2 ft.; 26 yds. 2 ft. and 23 yds. 1 ft.; 106 yds. 2 ft. and 93 yds. 1 ft.

III. Do. do.

27 yds. 2 ft. 6 in. and 23 yds. 0 ft. 2 in.; 41 yds. 2 ft. 3 in. and 34 yds. 1 ft. 9 in.; 13 yds. 2 ft. 9 in. and 11 yds. 1 ft. 7 in.; 25 yds. 2 ft. 8 in. and 23 yds. 1 ft. 4 in.; 38 yds. 2 ft. 6 in. and 35 yds. 0 ft. 6 in.; 12 yds. 2 ft. 10 in. and 11 yds. 2 ft. 2 in.

IV. Do. do.

680 and 635 yds.; 272 and 254 yds.; 408 and 381 yds.; 136 and 127 yds.

V. Do. do.

9 ch. 87 lks. and 7 ch. 89 lks.; 13 ch. 94 lks. and 7 ch. 75 lks.; 19 ch. 74 lks. and 15 ch. 78 lks.; 15 ch. 50 lks. and 6 ch. 97 lks.; 8 ch. 90 lks. and 8 ch. 09 lks.; 9 ch. 80 lks. and 9 ch. 08 lks.

Answers to Case I.

I. 143 yds.; 572 yds.; $331\frac{1}{2}$ yds.; $2983\frac{1}{2}$ yds.; $3321\frac{1}{2}$ yds.; $748\frac{1}{8}$ yds.; $83\frac{1}{8}$ yds.; $6733\frac{1}{8}$ yds.

II. 155 yds. 5 ft.; 2488 yds. 8 ft.; 622 yds. 2 ft.; 9955 yds. 5 ft.

III. 641 yds. 6 ft. 60 in.; 1443 yds. 7 ft. 99 in.; 160 yds. 3 ft. 123 in.; 606 yds. 8 ft. 80 in.; 1365 yds. 5 ft. 108 in.; 151 yds. 6 ft. 92 in.

IV. 89 a. 0 r. 34 p. 11½ yds.; 14 a. 1 r. 3 p. 27¼ yds.; 32 a. 0 r. 18 p. 23½ yds.; 3 a. 2 r. 10 p. 29½ yds.

V. 7 a. 7 ch. 8743 lks.; 10 a. 8 ch. 0350 lks.; 31 a. 1 ch. 4972 lks.; 10 a. 8 ch. 0350 lks.; 7 a. 2 ch. 0010 lks.; 8 a. 8 ch. 9840 lk.

Case II. Given, area and one side; find the other.

N.B. Length × Breadth = Area.

$$\therefore \text{ Length } - \quad - = \frac{\text{Area}}{\text{Breadth}}.$$

$$\text{And Breadth } - \quad - = \frac{\text{Area}}{\text{Length}}.$$

I. In the following oblongs, the area and one side are given, to find the other side.

	Area.	*Side* 1.	*Ans. Side* 2.
1.	572 sq yds.	26 yds.	22 yds.
2.	783 do.	29 do.	27 do.
3.	331½ do.	17 do.	19½ do.
4.	1461½ do.	37 do.	39½ do.
5.	2983½ do.	58½ do.	51 do.
6.	6733½ do.	78¾ do.	85½ do.

II. Do. do.

	Area.	*Side* 1.	*Ans. Side* 2.
1.	622 y. 2 f.	26 y. 2 f.	23 y. 1 f.
2.	2488 y. 8 f.	46 y. 2 f.	53 y. 1 f.
3.	9955 y. 5 f.	93 y. 1 f.	106 y. 2 f.
4.	160 y. 3 f. 123 in.	13 y. 2 f. 9 in.	11 y. 1 f. 7 in.
5.	1365 y. 5 f. 108 in.	38 y. 2 f. 6 in.	35 y. 0 f. 6 in.
6.	1443 y. 7 f. 99 in.	41 y. 2 f. 3 in.	34 y. 1 f. 9 in.

III. Do. do.

	Area.	*Side* 1.	*Ans. Side* 2.
1.	7 a. 0 r. 21 p. 28¾ y.	254 yds.	136 yds.
2.	14 a. 1 r. 3 p. 27¼ y.	544 do.	127 do.
3.	32 a. 0 r. 18 p. 23½ y.	190½ do.	816 do.
4.	15 a. 5 c. 7486 l.	9 c. 87 l.	15 c. 78 l.
5.	31 a. 1 c. 4972 l.	7 c. 89 l.	39 c. 48 l.
6.	5 a. 4 c. 0175 l.	7 c. 75 l.	6 c. 97 l.

Carpeting the Floor, and Papering the Walls, of a Room.

Fig. 4.

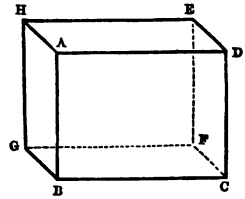
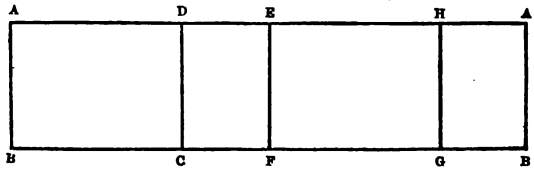


Fig. 5.



1. An ordinary room may be regarded as a box (*fig.* 4.); B C F G being the floor, A D E H the cieling *, A B C D and H G F E the side walls, and A B G H with D C F E the end walls.

2. Area of cieling = area of floor. (Hexagon, Note 5.)

3. The 4 walls of a room are usually oblongs, opposite sides being equal, and height the same all round. The total area, therefore, to be papered or painted, is the sum of the areas of the 4 walls,—the space occupied by windows, doors, fires, &c., being deducted when necessary.

4. Since area = length × breadth, it is usual, (to save

* Cieling from *ciel* (French), sky.

time and trouble in making estimates) to give dimensions in a corresponding manner: *e. g.*, if a room be 19 ft. long and 16 ft. wide, the area of its floor is at once written down as (19 × 16) ft. And if its height be 13 ft., then the area or surface of each wall will be (19 × 13) ft., or (16 × 13) ft., according as it lies along the length or breadth, *i. e.* along the side or end of the room.

Ex. 1. A room is 19 ft. long, 16 ft. wide and 13 ft. high. Windows, &c. occupy 153 sq. ft. Find wall surface to be papered.

Now, area of 2 side walls = (19 ft. × 13 ft.) × 2 = 494 sq. ft.
Do. 2 end do. = (16 ft. × 13 ft.) × 2 = 416 sq. ft.
∴ Total area of 4 walls, = 910 sq. ft.
Window area deducted = 153 sq. ft.
∴ Area to be papered = 757 sq. ft.

5. If the walls be ranged side by side, as in *fig.* 5., they form an oblong or rectangle, whose area = A A × A B = (70 × 13) ft. = 910 sq. ft., as before. This method is useful where the shape of the room is irregular. (Euc. II. 1.)

Ex. 2. The room in Ex. 1. is to be cieled, carpeted, and papered. It has 3 windows, each (3½ × 7) ft.; 2 doors, each (4 × 7½) ft.; and a fireplace (3 × 6½) ft. Cieling costs 4*s.* 6*d.* per sq. yd., Carpet 6*s.* 9*d.* do. and paper 2¼*d.* do. Find total cost.

Now area floor = (19 × 16) ft. = 304 sq. ft.
Do. cieling = (19 × 16) ft. = 304 sq. ft. } = 608 ft.

Also, area 4 walls = (*vid.* Ex. 1.) 910 sq. ft.
Area 3 windows = (3½ × 7) 3 = 73½ sq. ft.
Area 2 doors = (4 × 7½) 2 = 60 sq. ft. } = 153 sq. ft.
Area 1 fire = (3 × 6½) = 19½ sq. ft.

757 sq. ft. to be papered.

	£	s.	d.
Cost of cieling, 304 sq. ft. at 4*s.* 6*d.* per sq. yd. =	7	12	0
Cost of carpeting, 304 sq. ft. at 6*s.* 9*d.* per sq. yd. =	11	8	0
Cost of papering, 757 sq. ft. at 2¼*d.* per sq. yd. =	0	15	9¼
Total cost =	£19	15	9¼

6. Carpets and papers are seldom made yard wide, so that a yard long of either will not often be *a square yard*, and accordingly, the number of yds. required will seldom correspond with the number of sq. yds. in the floor or walls: *e. g.*, if the area of a floor be 30 sq. yds., while the carpet is only $\frac{3}{4}$ yd. or 27 in. wide, there will be required as many yards long of carpet as there are $\frac{3}{4}$ sq. yds. in the floor, viz. $(30 \div \frac{3}{4})$ or 40 yds. And if this carpet be 4*s.* 6*d.* per yd. long, the value of the whole flooring may be estimated as either (4*s.* 6*d.* × 40), or (6*s.* × 30), viz. £9 each way; the 6*s.* in the latter case being the value of a *square yard* of carpet at the same rate.

7. The value of such carpet or paper per sq. yd. is readily discovered thus: —

Ex. 1. Paper $\frac{3}{4}$ yd. wide is worth $2\frac{1}{2}d.$ per yd. long.
∴ Paper $\frac{1}{4}$ yd. wide is worth $\frac{1}{3}$ of $2\frac{1}{2}d.$ per yd. long; *i. e.* $\frac{5}{6}d.$
∴ Paper $\frac{4}{4}$ yd. wide (*i. e.* 1 sq. yd) is worth $(\frac{5}{6}d. \times 4)$ *i. e.* 3*d.*

Ex. 2. Carpet $1\frac{1}{4}$ or $\frac{5}{4}$ yd. wide is worth 7*s.* 6*d.* per yd long.
∴ Carpet $\frac{1}{4}$ yd. wide is worth $\frac{1}{5}$ of 7*s.* 6*d.* per yd. long, *i. e.* 1*s.* 6*d.*
∴ Carpet $\frac{4}{4}$ yd. wide (*i. e.* 1 sq. yd.) is worth (1*s.* 6*d.* × 4) *i. e.* 6*s.*

Ex. 3. Carpet 22 inches wide is worth 3*s.* $2\frac{1}{2}d.$ per yd. long.
∴ Carpet 1 inch wide is worth $\frac{1}{22}$ of 3*s.* $2\frac{1}{2}d.$ per yd. long, *i. e.* $1\frac{3}{4}d.$
∴ Carpet 36 inches (or 1 sq. yd.) is worth $(1\frac{3}{4}d. \times 36)$ *i. e.* 5*s.* 3*d.*

And all other widths may be treated in the same way:

1. Two rooms, *A* and *B*, measure $(23 \times 17\frac{1}{2})$ ft., and $(23\frac{1}{2} \times 17)$ ft. Find cost of carpeting each at 6*s.* 9*d.* sq. yd., and colouring each cieling at $4\frac{1}{2}d.$ do.

Ans. { *A.* £15 1*s.* $10\frac{1}{2}d.$, and 16*s.* $9\frac{1}{4}d.$
B. £14 19*s.* $7\frac{1}{2}d.$, and 16*s.* $7\frac{3}{4}d.$

2. Two rooms, *A* and *B*, measure $(25\frac{3}{4} \times 19)$ ft., and $(25 \times 19\frac{3}{4})$ ft. Find cost of carpeting each at 11*s.* 3*d.* sq. yd., and colouring each cieling at 1*s.* 6*d.* sq. yd.

Ans. { *A.* £30 11*s.* $6\frac{3}{4}d.$, and £4 1*s.* $6\frac{1}{2}d.$
B. £30 17*s.* $2\frac{1}{4}d.$, and £4 2*s.* $3\frac{1}{2}d.$

[*N.B.* The readiest mode of multiplying by $\frac{3}{4}$ is to take $\frac{1}{2}$ the quantity, and then $\frac{1}{4}$ of that again, and add them together.]

3. Two rooms, *A* and *B*, measure $(23\frac{1}{2} \times 17\frac{1}{2})$ ft., and $(25\frac{1}{2} \times 19\frac{1}{2})$ ft. Find cost of carpeting each at 3*s.* 9*d.* sq. yd., and colouring each cieling at 2*s.* 3*d.* sq. yd. (Vid. Qn. 5. note.)

Ans. { *A.* £8 11*s.* $4\frac{1}{4}$d., and £5 2*s.* $9\frac{3}{4}$d.
B. £10 7*s.* $2\frac{1}{4}$d., and £6 4*s.* $3\frac{3}{4}$d.

4. Two rooms, *A* and *B*, measure $(15\frac{1}{2} \times 13\frac{1}{2})$ ft. and $(19\frac{1}{2} \times 17\frac{1}{2})$ ft. Find cost of carpeting each at 3*s.* 6*d.* yd. ($\frac{3}{4}$ yd. wide), and papering each cieling at 2*d.* a yd.

Ans. { *A.* £6 2*s.* $0\frac{3}{4}$*d.*, and 5*s.* $9\frac{3}{4}$*d.*
B. £9 19*s.* $0\frac{3}{4}$*d.*, and 9*s.* $5\frac{3}{4}$*d.*

5. Two rooms, *A* and *B*, measure $(33\frac{1}{4} \times 28)$ ft., and $(28\frac{1}{4} \times 25\frac{1}{4})$ ft. Carpet *A* at 4*s.* $4\frac{1}{2}$*d.* a yd. ($1\frac{1}{4}$ yd. wide), and *B* at 5*s.* $7\frac{1}{2}$*d.* a yd. ($1\frac{1}{4}$ yd. wide.)

Ans. { *A.* £18 0*s.* $3\frac{9}{10}$*d.*
B. £18 3*s.* $8\frac{3}{10}$*d.*

N.B. The multiplication of fractional quantities, *as they stand*, may be readily effected by observing the algebraic method. Thus, to multiply $17\frac{1}{2}$ by $19\frac{3}{4}$.

Let $17 = a$ and $\frac{1}{2} = b$; then $17\frac{1}{2} = a + b$.
Also let $19 = c$ and $\frac{3}{4} = d$; then $19\frac{3}{4} = c + d$.

$$\begin{array}{r} a+b \\ c+d \\ \hline ac+bc \\ \quad ad+bd \\ \hline ac+bc+ad+bd \end{array}$$

Here observe that *both* the quantities *a* and *b* are multiplied by *c*, and *then* both by *d*, to avoid confusion.

Similarly:

$$\begin{array}{r} 17\frac{1}{2} \\ 19\frac{3}{4} \\ \hline 153 \\ 17 \\ \hline 323 = 17 \times 19 \\ 9\frac{1}{2} = \frac{1}{2} \times 19 \\ 13\frac{1}{8} = 17\frac{1}{2} \times \frac{3}{4} \\ \hline 345\frac{5}{8} = 17\frac{1}{2} \times 19\frac{3}{4} \end{array}$$

The same result, however, will be obtained, and often more readily, by decimals. *The two methods may be made to prove each other.*

6. Carpet and paper a room (39×23) ft. and 17 ft. high. It has 3 windows, each $(9\frac{1}{2} \times 8)$ ft.; 2 doors, each $(10\frac{1}{2} \times 6)$ ft.; 2 fires, each $(6\frac{1}{2} \times 4)$ ft. Carpet is 2*s.* 6*d.* a yd., 24 inches wide, and paper $1\frac{1}{2}$*d.* a yd. 24 inches wide. (*Vid.* notes 5 and 6.)

Ans. { Carpet £18 13*s.* 9*d.*
Paper £1 15*s.* $5\frac{1}{2}$*d.*

7. Carpet and paper a room (19×17) ft. and 13 ft. high. It has 2

windows (7×4) ft.; 2 doors ($9\frac{1}{2} \times 4$) ft.; 1 fire ($6\frac{1}{2} \times 4$). Carpet is 2*s.* 9*d.* a yd. 22 inches wide, and paper $2\frac{3}{4}$*d.* a yd., 22 inches wide.

Ans. { Carpet £8 1*s.* 6*d.*
Paper £1 12*s.* 5*d.*

8. Carpet and paper a room ($28\frac{1}{2} \times 22$) ft., and 14 ft. high. It has 6 windows ($9\frac{1}{2} \times 7$) ft.; 3 doors ($10\frac{1}{2} \times 6$) ft.; 2 fires ($5\frac{1}{2} \times 3$) ft. Carpet is 2*s.* $10\frac{1}{2}$*d.* a yd., 23 inches wide, paper $2\frac{1}{2}$*d.* a yd., 23 inches wide.

Ans. { Carpet £15 13*s.* 6*d.*
Paper £1 13*s.* $0\frac{1}{2}$*d.*

9. Carpet and paper a room ($29\frac{1}{2} \times 21\frac{1}{2}$) ft., and 16 ft. high. It has 5 windows ($5\frac{1}{2} \times 6$) ft.; 2 doors ($7\frac{1}{2} \times 9$) ft.; 3 fires ($6\frac{1}{2} \times 4$) ft. Carpet is 5*s.* 3*d.* a yd., 21 inches wide, and paper $1\frac{3}{4}$*d.* a yd., 21 inches wide.

Ans. { Carpet £31 14*s.* 3*d.*
Paper £1 14*s.* $10\frac{3}{4}$*d.*

10. Carpet and paper a room ($18\frac{1}{2} \times 16\frac{1}{2}$) ft., and 4 yds. high. It has 5 windows ($9\frac{1}{2} \times 8$) ft.; 2 doors ($10\frac{1}{2} \times 9$) ft.; 3 fires ($6 \times 3\frac{1}{2}$) ft. Carpet is 3*s.* $2\frac{1}{2}$*d.* a yd. 22 inches wide, and paper 3*d.* a yd., 48 inches wide.

Ans. { Carpet £8 18*s.* $0\frac{3}{4}$*d.*
Paper 4*s.* 4*d.*

11. Carpet and paper a room ($10\frac{1}{2} \times 9\frac{1}{2}$) yds. and $5\frac{1}{2}$ yds. high. It has 9 windows ($8\frac{3}{4} \times 5$) ft.; 3 doors ($9\frac{3}{4} \times 4$) ft.; 4 fires ($7\frac{1}{2} \times 3\frac{1}{2}$) ft. Carpet is 4*s.* $2\frac{3}{4}$*d.* a yd. 29 inches wide, and paper $2\frac{1}{2}$*d.* a yd. 46 inches wide.

Ans. { Carpet £25 5*s.* $3\frac{3}{4}$*d.*
Paper £1 6*s.* 7*d.*

12. Carpet and paper a room ($32\frac{1}{2} \times 21\frac{1}{2}$) ft., and 16 ft. high. It has 5 windows ($7\frac{1}{2} \times 6$) ft.; 3 doors ($9\frac{1}{2} \times 6$) ft.; 3 fires ($7\frac{1}{2} \times 4$) ft. Carpet is 6*s.* 5*d.* a yd. 22 inches wide, and paper $1\frac{3}{4}$*d.* a yd. 28 inches wide.

Ans. { Carpet £40 15*s.* $2\frac{1}{2}$*d.*
Paper £1 5*s.* $10\frac{1}{2}$*d.*

13. Carpet and paper a room ($32\frac{1}{2} \times 26$) ft., and 16 ft. high. It has 7 windows ($11\frac{1}{2} \times 8$) ft.; 4 doors ($10\frac{1}{2} \times 6\frac{1}{2}$) ft.; 3 fires ($7\frac{1}{2} \times 4$) ft. Carpet is 5*s.* $11\frac{3}{4}$*d.* a yd. 41 inches wide, and paper $2\frac{3}{4}$*d.* a yd. 44 inches wide.

Ans. { Carpet £24 12*s.* 11*d.*
Paper 18*s.* $\frac{1}{4}$*d.*

14. Carpet and paper a room ($31\frac{1}{2} \times 17$) ft. and 4 yds. 6 in. high. It has 3 windows ($5\frac{1}{2} \times 6$) ft.; 2 doors ($7\frac{1}{2} \times 4$) ft.; 2 fires ($3\frac{1}{2} \times 6\frac{1}{2}$) ft. Carpet is 10*s.* 6*d.* a yd. 27 inches wide, and paper $2\frac{1}{4}$*d.* a yd. 27 inches wide.

Ans. { Carpet £41 13*s.*
Paper £1 8*s.* 2*d.*

15. Carpet and paper 2 rooms, each ($19\frac{1}{2} \times 18\frac{1}{2}$) ft. and 14 ft. high. Each has 4 windows ($7\frac{1}{2} \times 5\frac{1}{2}$) ft.; 2 doors ($9\frac{1}{2} \times 5$) ft.; and 2 fires ($7\frac{1}{2} \times 3$) ft. Carpet is 2*s.* $7\frac{1}{4}$*d.* a yd. 25 inches wide, and paper $\frac{1}{2}$*d.* a yd. 42 inches wide.

Ans. { Carpet £15 0*s.* $7\frac{1}{2}$*d.*
Paper 10*s.* 6*d.*

16. How many yards long of carpet are used in questions 1, 2, 3, respectively? (Note 6.) *Ans.* $89\frac{1}{9}$; $109\frac{2}{3}$; $100\frac{17}{18}$.

17. How many yards of carpet and paper are used in questions 6, 7, 8 respectively?

Ans. { (6) Carpet $149\frac{1}{2}$ and paper $283\frac{3}{4}$; (7) $58\frac{6}{11}$ and $141\frac{5}{11}$; (8) $109\frac{1}{23}$ and $137\frac{11}{23}$.

18. How many yards of carpet and paper are used in questions 11, 12, 13 respectively?

Ans. { (11) Carpet $119\frac{28}{103}$ and paper $110\frac{32}{33}$; (12) $127\frac{1}{22}$ and $177\frac{3}{7}$; (13) $82\frac{126}{287}$ and $78\frac{7}{11}$.

Case II. To cover a floor or other area with planks, tiles, bricks, flags, slates, turfs, &c., *i. e.* to find *how many given smaller spaces will exactly fill up a larger one.*

N.B. Divide the greater space by the less, and the quotient is the number required.

Ex. 1. A square floor measures 17 ft. along one side; how many bricks (9 × 4) inches will cover it?

Here, area floor = (17 × 17) ft. = (17 × 17 × 144) inch.

Area brick = (9 × 4) inch. [and ∴ 36 times *fewer* bricks will be wanted than if each were only 1 sq. inch.]

$$\therefore \text{No. of bricks required} = \frac{17 \times 17 \times \overset{4}{\cancel{144}}}{\cancel{9} \times \cancel{4}} = 1156. \quad \textit{Ans.}$$

Ex. 2. A lawn is 75 yds. long and 54 wide. Find expence of laying it down with turfs each (15 × 9) inches, at 9*s.* per gross.

Area lawn = (75 × 54) yds. = (75 × 54 × 9 × 144) inches.

Area turf = (15 × 9) inches.

$$\therefore \text{No. of turfs required} = \frac{\overset{5}{\cancel{75}} \times 54 \times \cancel{9} \times 144}{\cancel{15} \times \cancel{9}} = 38880.$$

Here observe that primary law of arithmetic, viz., "find the value of *one*," which may be called the "rule of common sense," since it dispenses with all such puzzling contrivances as the Rule of Three, &c. &c., and proceed thus; —

Value of 144 turfs is 9*s.* or (9 × 12 × 4) farthings.

$$\therefore \text{Value of 1 turf is } \frac{9 \times 12 \times 4}{144} = \tfrac{3}{4}d.$$

∴ Value of 38,880 turfs is $\frac{3}{4}d. \times 38880 = £121\ 10s.$ *Ans.*

(*vid.* note to qn. 2 in "Carpet & Paper.")

Ex. 3. Tiles (9 × 4) inch being 41*s.* 8*d.* per 1000, it costs 48*s.* 2*d.* to *floor a square kitchen* with them. Find dimensions of kitchen.

Now value of one tile $= \frac{41s.\ 8d.}{1000} = \frac{1}{2}d.$; $\therefore$ as many tiles are used as there are halfpence in 48*s.* 2*d.* *i. e.* 1156.

$\therefore$ Area kitchen $= (36 \times 1156)$ sq. inches $= 41616$.

$\therefore$ Length of each side $= \sqrt{41616} = 204$ inches or 17 ft. *Ans.*

1. A roof (35×22) ft. is to be covered with slates (14×11) inches. How many are wanted? *Ans.* 6480.

2. A street just $\frac{1}{4}$ mile in length has on each side of it a footpath 7 ft. 6 inches wide. It is paved with flags each (2 ft. 9 in. $\times$ 1 ft. 8 in.) Find number used. *Ans.* 4320.

3. 4624 square tiles cover a sq. floor whose side is 5 yds. 2 ft. long. Find dimensions of a tile. *Ans.* 3 inches sq.

4. A wall space 4 yds. square is required to be divided into 64 equal compartments. Find dimensions of each. *Ans.* $1\frac{1}{2}$ ft.

5. A wall (23×13) yds. is to be built of stones $(16 \times 11\frac{1}{2})$ inches, outside face. Find cost of material at £6 5*s.* per 100. *Ans.* £131 12*s.* 6*d.*

6. A lawn is (72×49) yds. Turfs are (18×14) inches, and 1*s.* 3*d.* per doz. How much is overcharged by the contractor who sends in a bill of 100 guineas? *Ans.* 10 *gs.*

7. A room is 27 ft. 6 inches long and 8 yds. wide. Planks are 11 ft. long and 9 inches wide. How many are needed? *Ans.* 80.

8. An hotel-keeper gives orders for the construction of a racquet court $(18\frac{1}{2} \times 15\frac{1}{2})$ yds., to be laid with flags (31×18) inches, thinking the expence may be "about £50." Flags being £125 per 1000, how much out of his reckoning is he in this article alone? *Ans.* £33 5*s.*

9. Find cost of planking in Qn. 7 at 5*s.* $9\frac{3}{4}d.$ sq. yd. *Ans.* £21 6*s.* 3*d.*

10. A house has 24 windows, 11 of them $(3\frac{1}{2} \times 6\frac{1}{2})$ ft. and the rest $(2\frac{3}{4} \times 4\frac{1}{2})$ ft. each. The former are filled with panes $(13 \times 10\frac{1}{2})$ inches, and the latter with panes $(13\frac{1}{2} \times 8\frac{1}{4})$ inches each. How many of each will be required, and what will be the expense of the whole at 4*s.* 6*d.* a sq. yd.?

Ans. { 264 and 208. / £10 5*s.* $6\frac{3}{4}d.$

Miscellaneous Questions on the Square and Oblong.

N.B. The student is strongly recommended to draw neat and exact diagrams, whenever the subject is one which admits of such illustration. For not only is it an important matter to acquire facility as a draughtsman, which can only be done by careful practice, but it frequently happens that when the *eye* is thus brought to the assistance of the reasoning powers, perplexing questions at once become easy of

comprehension and solution, and difficulties vanish which before had seemed insuperable.

The only instruments requisite are a "scale of equal parts," "parallel ruler," "compasses, with pen and pencil," and "protractor," or brass half-circle (*vid.* Appendix) for measuring angles. These can all be bought of excellent quality for 3*s.*, at the National Society's Office, Westminster, or of Newton, the maker, Temple Bar.

1. How many plants, requiring each 3 sq. ft., will grow in a garden 19 yds. long and 16 yds. wide? *Ans.* 912.

2. How many sq. yds. are there in 19 yds. square? *Ans.* 361.

3. A garden is (17 × 16) yds. How many plants will grow in it, each requiring (1) 9 sq. inches, (2) 9 inches square?

Ans. (1) 39168; (2) 4352.

4. A piece of ground is (25 × 22) yds. Find its value at 3¾*d.* per square of 225 inches. *Ans.* £49 10*s.*

5. A room measures (180 × 162) inches. Find the number of sq. yds. in floor and cieling. *Ans.* 45.

6. How many persons can stand in a room (11½ × 9) yds., each occupying 931½ sq. inches? *Ans.* 144.

7. In a closely crowded room, 11 persons are counted along one end, and 15 along one side. Allowing a width of 23 inches to each person, find area of room, and number of people in it.

Ans. 67 yds. 3 ft. 21 in. About 165.

8. Troops marching in column form 324 ranks of 9 abreast. If these were drawn up in a solid square, how many would there be in each face?

Ans. 54.

[Illustrate this question with a diagram of 25, 36, &c., dots in square.]

9. And how many in the four faces? *Ans.* 212.

10. Show that a corps of 800 men will be formed by drafting from the above square the outside ranks, 4 deep, all round.

11. A drawing-room is (23 × 18) ft., and dining-room (21 × 17) ft. Find the expense of carpeting the former with Brussels at 10*s.* 6*d.* per yd. (¾ yd. wide), and the latter with Axminster at 7*s.* 6*d.* per yard (¾ yd. wide.) *Ans.* £32 4*s.* and £19 16*s.* 8*d.*

12. A drawing-room is (22 × 18) ft., and dining-room (21 × 15) ft. Find the expense of carpeting the former with Axminster at 7*s.* 4½*d.* per yd. (27 inches wide) and the latter with Kidderminster at 4*s.* 7½*d.* per yd. (27 inches wide). *Ans.* £21 12*s.* 8*d.* and £10 15*s.* 10*d.*

13. 188 sq. ft. are to be cut from a plank 23½ inches wide. What length must be taken? *Ans.* 96 ft.

14. 97,535,376 carrots, each occupying 9 sq. inches, grow in a sq. field. Find its dimensions. *Ans.* 823 yds. sq.

15. 94,249 soldiers are drawn up in a solid square. How much ground will they cover if each occupies a space of 27 inches sq., and what will be the length of one side?

Ans. 4 a. 3 r. 18 p. $27\frac{3}{4}$ yds. and $153\frac{1}{2}$ yds.

16. A field is (13 × 9) poles. How many roods and sq. rods does it contain? *Ans.* 2 roods and 37 rods.

17. A floor (25 × 35) ft. is laid with planks $17\frac{1}{2}$ ft. × 15 inches. Find value of flooring at 13*s.* 6*d.* per sq. yd., and number of planks.

Ans. £65 12*s.* 6*d.*; 40.

18. A chess-board is a foot square. Find dimensions of each of its 64 equal squares. *Ans.* $1\frac{1}{2}$ inch.

19. Room *A* measures $(17\frac{1}{2} \times 15)$ ft., and *B* $(13\frac{1}{2} \times 13\frac{1}{2})$ ft. Which will cost more to carpet at 6*s.* 9*d.* per sq. yd., and how much? *Ans.* *A*, 40*s.* 6*d.*

20. How many boards, each 11 ft. long and 13 inches wide, will be required for the flooring of a room which is 8 yds. 2 ft. × 9 yds. 0 ft. 6 in.? And what will they cost at 4*s.* $3\frac{3}{4}$*d.* per sq. yd.?

Ans. 60, and £17 2*s.* $7\frac{1}{4}$*d.*

21. Window *A* is 3 panes wide and 4 high, *B* is 4 wide and 5 high. Each of *A's* panes are (15 × 9) inches, and each of *B's* (13 × 8) inches. What will be the expense of glazing each window at 9*d.* per sq. ft.?

Ans. *A*, 8*s.* $5\frac{1}{4}$*d.*; *B*, 10*s.* 10*d.*

22. A garden contains $423\frac{1}{2}$ sq. yds. One side measures exactly a chain. Show that the other is exactly $3\frac{1}{2}$ poles long.

23. A court (9 × 7) yds. is to be paved with either bricks or tiles. Bricks are (9 × 3) inches, and 20*s.* 10*d.* per 1000; while tiles are 6 inches square and 3*s.* 9*d.* per gross. Which will be cheaper, and how much?

Ans. Tiles, $5\frac{1}{4}$*d.*

24. Two gardens, *A* and *B*, (28 × 24) yds. and (23 × 18) yds. respectively, are each made 4 yds. longer. How much must be cut off from the width (*i. e. shorter side*) of each, so that their areas may remain unaltered? *Ans.* *A*, 9 ft., *B*, 8 ft.

25. How much will a person lose who undertakes to fence all round, at 13*s.* 9*d.* per rod, both the above gardens for the same money *after* the alteration as *before*? *Ans.* 11*s.* 8*d.*

26. A barn (38 × 29) ft. is plastered inside and out to the height of 19 ft., at 6*s.* 3*d.* per "square" of 100 ft. Find cost. *Ans.* £7 19*s.* $1\frac{1}{2}$*d.*

27. The two roofs of a house measure (38 × 21) ft. and (38 × 18) ft. Find cost of slates (19 × 14) inches, at £7 5*s.* 10*d.* per 1000.

Ans. £5 17*s.*

28. A street $\frac{3}{4}$ mile long and 35 ft. wide, is to be paved with granite blocks (11 × 3) inches upper surface. The material is contracted for at £3150. How much is this per 1000? Ans. £5 4*s.* 2*d.*

29. Draw neat diagrams showing accurately that 9 sq. ft. = 1 sq. yd.
$\sqrt{25} = 5$; $(\frac{3}{4})^2 = \frac{9}{16}$; $\sqrt{\frac{16}{25}} = \frac{4}{5}$.

30. In how many shapes may a piece of ground be traced out so as always to contain 48 full sq. yds. ?

Ans. 5 ; viz. 1×48, 2×24, 3×16, 4×12, 6×8 yds.

31. How many persons can have allotments of $2\frac{1}{2}$ acres each out of a common containing $2\frac{1}{2}$ sq. miles ? *Ans.* 640.

32. 58 ropes measure each 689 yds. 2 ft. $9\frac{1}{2}$ in.
76 do. do. 985 yds. 1 ft. $11\frac{1}{4}$ in.
68 do. do. 473 yds. 2 ft. $7\frac{3}{4}$ in.
Find total and average length, and value of the whole at $4\frac{1}{2}d.$ per yd.

Ans. { Total 147149 yds. 0 ft. 1 in.
Average 728 yds. 1 ft. $4\frac{117}{203}$ in.
Value £2759 0*s.* $10\frac{5}{8}d.$

N.B. If the contributions to the total be massed together and then divided equally among the contributors, this equal share is called the *Average*. Thus, if in a class of 15 boys 4 have 1*s.* 6*d.* each, 5 each 2*s.*, and the rest 3*s.* 7*d.* each, their total wealth amounts to 37*s.* 6*d.*, and this, divided equally among the fifteen, gives the *average share of each*, viz. 2*s.* 6*d.* ; *i. e.* the *same total* of 37*s.* 6*d.* is arrived at, whether the class share it equally, or in unequal portions as above.

33. 34 rolls of calico measure each 947 yds. 2 ft. $3\frac{1}{2}$ in. in length.
38 do. do. 1971 yds. 0 ft. $10\frac{1}{2}$ in. in length.
116 do. do. 344 yds. 2 ft. $10\frac{3}{4}$ in. in length.
Find total and average length, and value of whole at $6\frac{3}{4}d.$ per yd.

Ans. { Total 147149 yds. 0 ft. 1 in.
Average 782 yds. 2 ft. $1\frac{89}{180}$ in.
Value £4138 11*s.* $3\frac{15}{16}d.$

34. 23 rooms contain each 79 sq. yds. 8 ft. 132 in.
34 do. do. 48 sq. yds. 7 ft. $79\frac{1}{2}$ in.
47 do. do. 68 sq. yds. 6 ft. 86 in.
Find total and average contents of floors, and value of carpet for the whole at 13*s.* 6*d.* per sq. yd.

Ans. { Total 6730 yds. 6 ft. 133 in.
Average 64 yds. 6 ft. $67\frac{77}{104}$ in.
Value £4543 5*s.* $4\frac{5}{8}d.$

35. 76 fields contain each 2 a. 3 r. $29\frac{3}{4}$ p.
68 „ „ 3 a. 0 r. $39\frac{1}{2}$ p.
39 „ „ 5 a. 2 r. $27\frac{1}{4}$ p.
Find total and average area, and value of whole at £60 per acre.

Ans. { Total 665 a. 0 r. $9\frac{3}{4}$ p.
Average 3 a. 2 r. $21\frac{1}{4}$ and $\frac{84}{183}$ p.
Value £39903 13*s.* $1\frac{1}{2}d.$

36. 186 fields contain each 4 a. 1 r. 19¼ p.
379 „ „ 3 a. 3 r. 17¾ p.
274 „ „ 2 a. 2 r. 26½ p.
Find total and average area, and value of whole at £8 15*s.* per rood.

Ans. { Total 3006 a. 2 r. 8¾ p.
Average 3 a. 2 r. 13¼ and $\frac{368}{839}$ p.
Value £105229 8*s.* 3¾*d.*

37. 26 fields contain each 1 a. 9 c. 9875 lks.
37 „ „ 2 a. 7 c. 4653 lks.
43 „ „ 3 a. 3 c. 3333 lks.
Find total and average area, and value of whole at £104 3*s.* 4*d.* per acre.

Ans. { Total 296 a. 9 c. 2230 lks.
Average 2 a. 8 c. 115$\frac{32}{33}$ lks.
Value £30929 8*s.* 1½*d.*

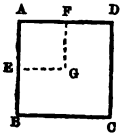
38. 78 fields contain each 1 a. 7 c. 8567 lks.
49 „ „ 2 a. 5 c. 9876 lks.
56 „ „ 6 a. 9 c. 8887 lks.
Find total and average area, and value of whole at £5 4*s.* 2*d.* per chain.

Ans. { Total 657 a. 9 c. 9822 lks.
Average 3 a. 5 c. 9561$\frac{53}{61}$ lks.
Value £34270 14*s.* 9¾*d.*

39. A gentleman had a square window in his study, a yard high and a yard wide. Finding it too large, he had exactly half of it removed, and still had a square window a yard high and a yard wide. Show by diagram how this was done.

40. A father who had 5 sons divided a square field of 32 acres among them as follows:—The eldest had a square of 8 acres, viz. A E F G (*vid. fig.*) leaving a Gnomon (*vid.* Euclid, bk. ii.) to be divided equally among the other 4, who had each 6 acres in the shape of a gnomon of exactly the same character as the original. Show by diagram how the division was effected.

Fig. 6.



41. A railway of 22½ miles is laid with rails weighing 12 lbs. to the foot. Find cost of a *double line* of rails at £7 per ton. *Ans.* £17820.

42. A regiment of 789 men receive coats, each containing 2½ yds. cloth, (1¼ yd. wide), and 2½ yds. lining (¾ yd. wide). Find total cost of coats, cloth being 12*s.* 6*d.* and lining 2*s.* 9*d.* per sq. yd.
Ans. £1347 17*s.* 6*d.*

43. A piece of land is (179 × 148) yds. Find its value at £60 10*s.* per acre, and cost of fencing all round at 13*s.* 9*d.* per rod.
Ans. £331 3*s.* and £81 15*s.*

44. A square field contains 97535376 sq. inches. Find cost of fencing its four sides at 1*s.* 11½*d.* per yd. Ans. £106 6*s.* 1*d.*

45. One side of a sq. field is $9\frac{1}{2}$ rods. Find its value at £20 3*s.* 4*d.* per rood, and cost of fencing 3 sides at 1*s.* 6*d.* per yd.

Ans. £45 10*s.* $0\frac{1}{4}$*d.* and £11 15*s.* $1\frac{1}{2}$*d.*

46. A square Turkey carpet, $5\frac{1}{2}$ yds. each way, partially covers a floor (23 × 19) ft. What will it cost to complete the flooring with oilcloth at 4*s.* 6*d.* per yd. $\frac{2}{4}$yd. wide? *Ans.* £5 9*s.* 10*d.*

47. How many tiles $(9 \times 4\frac{1}{2})$ inches will pave a floor $(97\frac{1}{2} \times 81)$ ft., and what will they cost at 9*s.* per gross? *Ans.* 28032 and £87 12*s.*

48. Sides of squares *A* and *B* are 45 and 33 rods respectively. How much more is *A* worth than *B*, at the rate of £80 13*s.* 4*d.* per acre?

Ans. £471 18*s.*

49. $73\frac{3}{4}$ acres are for sale. *A* offers 5*d.* per sq. yard, and *B* $\frac{1}{4}$*d.* per sq. link, for the lot. Which offer is the best for the seller, and how much? *Ans.* *B*, £245 16*s.* 8*d.*

50. A lawn (35 × 27) yds. is to be laid down with turfs (21 × 18) inches. Find number required, and cost of the whole at 6*s.* 3*d.* per 100.

Ans. 3240 and £10 2*s.* 6*d.*

51. Side of square *A* is 17 yds. 2 ft. 9 in. and side of square *B* is just double this length. Find no. of sq. inches by which *B* exceeds *A*.

Ans. 1248075.

52. (A.) A square court whose side is 34 yds. is paved with 18496 sq. tiles. Find dimensions of each. *Ans.* 9 inches sq.

52. (B.) Two square courts require 4 times and $\frac{1}{4}$ times respectively, as many tiles as in 52 (A.) Find length of each court.

Ans. 68 and 17 yds.

52. (C.) *M* and *N* are two gardens (60 × 80) yds. and (40 × 50) yds. Area of *M* is to be doubled, and *N* trebled, without in either case altering the relative proportion of the sides; *i. e.* breadth and length are still to be as 3 : 4 in *M*, and as 4 : 5 in *N*; or, which is the same thing, breadth is still to be $\frac{3}{4}$ length in *M*, and $\frac{4}{5}$ length in *N*. Find new dimensions. (*Vid.* Notes 14—16.)

Ans. { *M*, (84·852 × 113·136) yds. *N*, (69·28 × 86·6) yds.

53. Two fields need fencing. *A* contains 14356521 sq. inches, and *B* 129208689 do. Find cost of fencing all four sides of each at 12*s.* $4\frac{1}{2}$*d.* per rod.

Ans. { £ 47 7*s.* 3*d.* £142 1*s.* 9*d.*

54. Oilcloth is 2*s.* $1\frac{1}{2}$*d.* per yd. (27 inches wide), while carpet is $3\frac{3}{4}$*d.* per sq. ft. How much cheaper will the latter be for a floor (27 × 23) ft.?

Ans. 1*s.* $5\frac{1}{4}$*d.*

55. A room is $(28\frac{1}{4} \times 25\frac{1}{4})$ ft. Carpet $(1\frac{1}{4}$ yd. wide) is 5*s.* $7\frac{1}{2}$*d.* per yd. Find quantity required and cost. *Ans.* $64\frac{3}{4}$ yds. nearly, and £18 3*s.* $8\frac{3}{4}$*d.*

56. A piece of land $(1\frac{1}{2} \times \frac{3}{4})$ miles is to be planted with shrubs at

27*s.* per gross, each requiring 5 ft. sq. How many must be ordered, and what will be the cost? *Ans.* 1254528 and £11761 4*s.*

57. A barn is (59 × 28) ft. Find cost of flooring it with tiles (13 × 9) inches, at 1*s.* 3*d.* per score, and of plastering the walls inside and out, to the height of 13 ft., at 1*s.* 8½*d* per sq. yd.

Ans. £6 7*s.* $0\frac{1}{10}$*d.* and £42 8*s.* 3*d.*

58. A square field contains 78965 sq. yds. Find cost of fencing all round at $9\frac{3}{4}$*d.* per ft. *Ans.* £136 19*s.* 9½*d.*

59. Find the number of yds. required for a room ($27\frac{5}{8}$ × $29\frac{1}{16}$) ft. And its cost at 5*s.* 7½*d.* per yd. ($1\frac{1}{4}$ yd. wide.) *Ans.* £20 10*s.* $0\frac{55}{61}$*d.*

60. Square *A* contains 213 yds. 4 ft. 52 in.; *B*, $286\frac{35}{144}$ inches; and *C*, 195 yds. $3\frac{23}{256}$ ft. Find a side of each.

Ans. A, 526 in., *B*, $16\frac{1}{12}$ yds., and *C*, $41\frac{1}{16}$ ft.

61. From one acre of ground which is filled with shrubs occupying each 6 sq. ft., plants are taken to stock another acre where each will occupy 6 ft. sq. At £4 10*s.* per gross, what is the value of the shrubs taken, and also of those left? *Ans.* £226 17*s.* 6*d.* and £1134 7*s.* 6*d.*

62. A piece of road, $5\frac{1}{2}$ miles long, and 29 ft. wide, is to be stoned at 2½*d.* per sq. yd. Contractor *A* tenders for half of it at £8500, and *B* for the other half at £9500. What will each gain or lose by his contract?

Ans. { *A* loses £272 10*s.*
B gains £727 10*s.*

63. A window of 5 lights is 17 ft. high and 14 ft. wide. Its 4 stone mullions are each $3\frac{1}{2}$ inches thick, (or wide). What will it cost to fill the whole window with painted glass at 3*s.* 6*d.* per sq. ft.?

Ans. £38 3*s.* 7*d.*

64. How much space, at $7\frac{1}{2}$ sq. ft. per man, will be occupied by a solid square of infantry, having 93 men in each face? And what will it cost to fit out the outside ranks, all round 3 deep, at £23 15*s.* 6*d.* per man?

Ans. { 7207½ sq. yds.
£25677.

[*N.B.* Corners not counted twice. Each rank all round numbers 8 fewer than the one just in front of it. One man stands alone in the centre of an *odd* square, but 4 together in that of an *even* one. The student should realise this to himself by constructing squares with 5, 6, &c., dots in each face.]

65. How much space at $5\frac{1}{3}$ sq. ft. each man, 97 in each face? And cost of outside ranks, all round 5 deep, at $22\frac{1}{2}$ guineas per man?

Ans. { 5749 yds. 8 ft. 72 in.
£43470.

66. How much space by a solid cavalry square, $32\frac{3}{4}$ sq. ft. each, 85 in each face, and cost of equipping outside ranks, 7 deep, at $49\frac{1}{2}$ gs. per man?

Ans. { $26290\frac{35}{36}$ sq. yds.
£1135113 8*s.*

67. Two solid squares A and B number 24 and 25 men respectively in each face. If both are made hollow by the removal of a square central body, each side of which numbers 7 ranks in depth to its centre inclusive, how many will have been withdrawn from each, and how many ranks and men left? (*Vid.* note, Qn. 64.)

Ans. { Withdrawn, A, 196; B, 169.
Left, A, 5 ranks, and 380 men.
Left, B, 6 ranks, and 456 men.

68. Two solid squares A and B number 34 and 35 men respectively in each face. If both are made hollow by the removal of a square central body, each side of which numbers 11 ranks in depth to its centre inclusive, how many will have been withdrawn from each, and how many ranks and men left?

Ans. { Withdrawn, A, 484; B, 441.
Left, A, 6 ranks, and 672 men.
Left, B, 7 ranks, and 784 men.

69. Troops are drawn up in column, 54 in front, and 96 in flank. Show that their formation can be changed to a solid square, and that there will then be 60 men fewer in the outside ranks, 5 deep all round, than there were in the outside ranks of the column.

70. How much ground will they cover in each case, allowing $6\frac{3}{4}$ sq. ft. to each man? *Ans.* 3888 sq. yds.

71. What will be the length and breadth of this ground when occupied by column and square respectively?

Ans. { *C.* 83·136 yds. and 46 yds. 2 ft. $3\frac{1}{2}$ in.
S. 62·353 yds.

72. In how many different ways may a corps of 360 men be drawn up?

Ans. 12. { viz. 1×360; 2×180; 3×120; &c.
i. e. single line; then 2, 3, &c., deep.

73. Find the number of men in each face of the 3 following solid squares; viz.

A. Each man covers $4\frac{1}{2}$ sq. ft. and whole square $3612\frac{1}{2}$ yds.
B. " $5\frac{1}{2}$ " $5749\frac{17}{18}$ yds.
C. " $7\frac{1}{2}$ " $7207\frac{1}{2}$ yds.

Ans. A, 85; B, 97; C, 93.

74. On attempting to form a body of soldiers in solid square, it was found that there were 82 over; and yet, that the square's face could not be increased 1 man, without an additional force of 83. How many were there? *Ans.* 6806.

[*N. B.* It is well worth noticing, as an important fact, and one that will often facilitate calculation, that "*The difference of the squares of any two consecutive numbers equals the sum of those numbers;*" *e. g.* $4^2-3^2=4+3$; *i. e.* $16-9=7$. And so on. How this arises may be easily and clearly seen by drawing a diagram in which 16 dots are ranged as a square, and then removing dots from the outside rows until 9 remain as

a square. Similarly, it will be found that $(4+5)$ dots must be added to the 16 in order to make a square having five in each face.]

75. On attempting to form a body of soldiers in solid square, it was found that there were 54 over; and yet, that the square's face could not be increased 1 man without an additional force of 57. How many were there? *Ans.* 3079.

76. On attempting to form a body of soldiers in solid square, it was found that there were 75 over; and yet, that the square's face could not be increased 1 man without an additional force of 16. How many were there? *Ans.* 2100.

[*N.B.* To solve the three foregoing questions. Take No. 74.

Let x = No. in face of 1st square;

Then $x^2 + 82$ = total no. of men by Qn.

Now by Qn., $x^2+82+83 = (x+1)^2 = x^2+2x+1$.

$\therefore 2x = 82+83-1$, (for x^2 goes out)

$$\therefore x = \frac{164}{2} = 82;$$

And $\therefore 82^2+82$ or 6806 = No. required.]

77. Find the expense, at $15\frac{3}{4}$ gs. per man, of equipping the outside ranks, 4 deep all round, (1.) of the column, (2.) of the square, in Qn. 69. *Ans.* £18786 12*s.*; £17992 16*s.*

78. A troop of 560 men is formed by drafting off the outside ranks, 2 deep all round, from a solid square. Find the number in each face of the square before and after the draft. (*Vid.* note, question 64.)

Ans. 72 and 68.

79. A troop of 504 men is formed by drafting off the outside ranks, 3 deep all round, from a solid square. Find the number in each face of the square before and after the draft. *Ans.* 45 and 39.

80. A troop of 528 men is formed by drafting off the outside ranks, 4 deep all round, from a solid square. Find the number in each face of the square before and after the draft. *Ans.* 37 and 29.

81. A troop of 1240 men is formed by drafting off the outside ranks, 5 deep all round, from a solid square. Find the number in each face of the square before and after the draft. *Ans.* 67 and 57.

[*N.B.* To solve the foregoing four questions. Take No. 78.

Let x=number in each face at first.

Then $\{2x+2\,(x-2)\}$ or $(4x-4)$=number in 4 faces.

And $(4x-4)-8$=number in 2nd rank all round. (Note question 64.)

Now by question, $(4x-4)+\{4x-4)-8\}=560$.

Or $2\,(4x-4)-8=560$.

$\therefore x=72$.

The above may be readily illustrated by a dotted diagram, exhibiting a solid square with 4, 5, or 6, &c. dots in each face.]

82. From a hollow square, 3 deep and 76 in each front, 12 men are

withdrawn, and the rest formed into a square 4 deep. How many are there in each face of the second square? *Ans.* 58.

83. Carpet 30 inches wide is 2*s.* 6*d.* per yd., while matting is $3\frac{3}{4}d.$ per sq. ft. Which will be cheaper, and how much, for a floor ($23\frac{1}{2} \times 18$) ft.?

Ans. Matting, 8*s.* $9\frac{3}{4}d.$

84. Oilcloth $\frac{3}{4}$ yd. wide is 2*s.* 3*d.* a yd., while carpet is $3\frac{3}{4}d.$ a sq. ft. What will be saved by using the latter for a floor 19 ft. 9 in. × 16 ft.?

Ans. 6*s.* 7*d.*

85. Gardens *A* and *B* are (40 × 30) yds. and (96 × 72) yds. *A* is to be lengthened 10 yds., and *B* 12 yds. How much must the width of each be diminished, so that their areas may remain unchanged?

Ans. 6 and 8 yds.

86. (A.) How much out of pocket will the mason be who contracts, at 16*s.* 6*d.* a rod, to wall both *A* and *B* for the same money *after* as *before* the alteration? *Ans.* £2 8*s.*

86. (B.) What alteration will be effected in the area of these gardens, if the width of *A* become 15 yds., and of *B* 144 yds.?

Ans. Halved; doubled.

86. (C.) It is required to make *A* 5, and *B* 6 times its present size, by increasing both length and breadth *proportionally*. What addition will have to be made to both, in each instance? (*Vid.* question 52. C.)

Ans. { *A*, 49·44 and 37·08 yds.
B, 139·152 and 104·364 yds.

87. The area of a floor is 588 sq. ft., and its length and breadth are in the ratio of 4 : 3. Find them. *Ans.* 28 and 21 ft.

88. The area of a floor is 540 sq. ft. Breadth is $\frac{3}{5}$ length. Find them.

Ans. 6 and 10 yds.

89. The area of a floor is 142 yds. 2 ft., and 5 times the breadth = 4 times the length.

[*N. B.* To solve the three foregoing. Take No. 87.

Let $4x$ ft. = length; then $3x$ ft. = breadth.

Now area = $(4x \times 3x)$ or $12x^2 = 588$, by question.

$\therefore x^2 = 49$ and $x = 7$ &c.]

90. How many days of $8\frac{3}{4}$ hours will it take to dig a ditch all round a field containing 360000 sq. ft. at rate of $2\frac{1}{2}$ yds. in $1\frac{3}{4}$ hours?

Ans. 64 days, *i. e.* 10 weeks and 4 days.

91. Five fields contain 28 a. 2 r. $24\frac{1}{2}$ p. altogether. Four of them contain respectively, 3 a. 2 r. $37\frac{3}{4}$ p., 4 a. 3 r. $29\frac{1}{4}$, 5 a. 1 r. $18\frac{3}{4}$ p. and 6 a. 3 r. $33\frac{3}{4}$ p. Find value of fifth field and of the whole, at 7*s.* 6*d.* per sq. rod. *Ans.* £459 5*s.* $7\frac{1}{2}d.$ and £1719 3*s.* 9*d.*

92. A father divided 300 acres between his 8 sons in proportion to their ages, viz., 20, 15, 10, 8, 5, 3, 2, 1. Find share of each.

Ans. 93 a. 3 r.; 70 a. 1 r. 10 p.; 46 a. 3 r. 20 p.; 37 a. 2 r.; 23 a. 1 r. 30 p.; 14 a. 0 r. 10 p.; 9 a. 1 r. 20 p.; 4 a. 2 r. 30 p.

93. Five fields are to be fenced. *A* is (415 × 297) ft., *B* (291 × 341) ft., *C* (173 × 223) ft., *D* (314 × 201) ft., and *E* (215 × 213) ft. Find cost of fencing them all round, (1.) singly, (2.) in one square block, at 8*s.* 3*d.* a rod. *Ans.* £134 3*s.* and £60 16*s.* 6*d.*

94. Four fields measure as follows:—*A* (517 × 298) ft., *B* (369 × 642) ft., *C* (349 × 705) ft., and *D* (357 × 478) ft. Find total cost of fencing each at 12*s.* 4½*d.* a rod. *Ans.* £278 12*s.* 6*d.*

95. Find total acreage of the fields in Qns. 93. and 94. respectively.

Ans. { 93. 8 a. 1 r. 38 p. 28 yds. 6½ ft.
94. 18 a. 2 r. 6 p. 17 yds. 8½ ft.

96. The sides of 3 squares are 4, 5, 6, yds. respectively. Find the side of another which will exactly contain all three. *Ans.* 8·7749 yds.

97. Room *A* is (17½ × 15) yds. and *B* (12 × 9½) yds. The former is filled with persons occupying 6¾ sq. ft. each, the latter with persons occupying 4½ ft. Admission to *A* is 2*s.* 6*d.*, to *B* 3*s.* 9*d.* Find total receipts in each. *Ans. A* £43 15; *B* £42 15.

98. If the expense of an evening's entertainment in the above rooms be £58 6*s.* 8*d.* and £57 respectively, and the same area be allowed to each person, what is the least sum that each must pay, so as to make up this outlay? *Ans. A*, 3*s.* 4*d.*; *B*, 5*s.*

99. It is proposed to give a concert in a room (21 × 16) yds. of which area ⅜ths at one end are occupied by the orchestra. Benches 15 inches wide are placed for the audience across the room, 2 ft. 3 in. apart, leaving a passage of 4 ft. wide up the centre. Allowing 22 inches of bench to each person, how many can be accommodated? *Ans.* 336.

100. If the front 7 rows in the above room pay half as much again as those behind, and the expenses of the concert be 100 gs., what is the least that each must pay to make it up? And what, on the supposition that the room will be only ⅔rds full?

Ans. (1.) 7*s.* 6*d.* and 5*s.*; (2.) 11*s.* 9*d.* and 7*s.* 6*d.*

101. A procession consists of persons walking 7 abreast. Each row extends 5 yds. in width, and from front to front of any two consecutive rows is 1½ yds. At the rate of 3 miles an hour it is 15 minutes in passing any point on the road. Find number of persons, and space occupied. *Ans.* 6160 men and 6600 sq. yds.

102. A picture measures (32½ × 26½) inches *with* the frame, and (29 × 23) *without*. How many square inches of gold leaf will be needed for the frame? *Ans.* 194¼.

103. A picture-gallery is just 3 times as long as it is broad. The carpeting, at 6*s.* 9*d.* sq. yd., costs £40 12*s.* 3*d.*, and papering the whole wall area at 2¼*d.* sq. yd. is £2 13*s.* 10*d.* Find dimensions of gallery.

Ans. (57 × 19 × 17) ft.

104. A garden containing 2304 sq. yds. is 4 times as long as it is

broad. Its sides and ends are joined by 2 straight walks, $4\frac{1}{2}$ ft. wide, crossing each other at the centre. Find space left for planting.

Ans. $2126\frac{1}{4}$ yds.

105. A roller is $2\frac{1}{2}$ ft. long, and $7\frac{1}{2}$ ft. in circumference. From end to end of a bowling-green it makes 28 revolutions, and requires to pass 56 times up and down before the whole surface is rolled once. Find area of green. *Ans.* $3266\frac{2}{3}$ yds.

106. A garden (94 × 68) yds. has a walk $4\frac{1}{2}$ ft. wide all round, close to the wall. Its sides and ends are also connected by 2 other walks of the same width crossing at right angles in the middle. Find the space to be gravelled, and cost at $2\frac{1}{2}d.$ per sq. yd.

Ans. { $708\frac{3}{4}$ sq. yds. / £6 12*s.* $10\frac{1}{16}d.$

106. (B.) Three gardens measure (65 × 45) yds. each. It is proposed to take $\frac{1}{5}$ from *A*, $\frac{1}{3}$ from *B*, and $\frac{1}{9}$ from *C*, still leaving the sides of each in the same proportion of 13 : 9 as at present. Find new dimensions of each.

Ans. { *A*, 53·07315 × 36·743 yds. / *B*, 45·9615 × 31·8195 yds. / *C*, 37·528 × 25·98 yds.

106. (C.) If, however, instead of being lessened, *A* were made 5 times, *B* 6 times, and *C* 7 times as large proportionally, find expense of fencing each at 4*s.* $1\frac{1}{2}d.$ per rod.

Ans. { *A*, £18 8*s.* $11\frac{1}{4}d.$ / *B*, £20 4*s.* 1*d.* / *C*, £21 16*s.* $5\frac{1}{16}d.$

107. A shepherd having 102 hurdles at his disposal, used 100 of them in making a rectangular fold for half his flock, *so* that, if needful, it would hold the other half too, with merely the addition of the 2 spare hurdles. Draw a diagram showing how it was contrived. (*Vid.* "Square," note 13. &c.)

108. A gentleman had a square cellar, with a wine-bin in each corner and one in the middle of each side. The corner ones contained 1 dozen each, and the middle ones 7 dozen each, so that 9 dozen could always be counted on each side. A dishonest butler on 3 successive occasions removed 4 dozen (*i. e.* 12 dozen in all), and yet each time contrived so to arrange the rest that 9 dozen could always be counted on each side. How was it done? (*Vid.* note, question 64.)

109. I have a square table of such dimensions that the number of ft. in the length of its 4 sides is just $\frac{3}{4}$ the number of sq. ft. in its surface. Find length of side. *Ans.* 2 yds.

110. A garden is 46 yds. wide, and if it were 12 yds. longer it would contain 2 r. 26 p. $13\frac{1}{2}$ yds. How long is it? *Ans.* 58 yds.

111. A meadow contains exactly an acre, and the difference of its adjacent sides is 33 yds. Find them. *Ans.* 55 and 88 yds.

112. A garden contains 2 r. 15 p. $6\frac{1}{4}$ yds., and width is $\frac{5}{6}$ length. Find them. *Ans.* 48 and 60 yds.

113. Length of a field is 3 times breadth; but when both are diminished 40 yds., length is 4 times breadth. Find dimensions.

Ans. 360 and 120 yds.

114. A square garden is exchanged for an oblong of the same area, which is 16 yds. longer and 12 yds. shorter than the square. What is its area? *Ans.* 2304 yds.

115. The area of a picture is 360 inches; but when length and breadth are diminished 2 inches by the removal of its inch-wide frame its area is only 288 inches. Find dimensions. *Ans.* 20 and 18 inches.

116. Flags ($3 \times 2\frac{1}{2}$) ft. are used for the pavement on both sides of a street $\frac{1}{8}$ mile long. Had they been 3 ft. sq., 220 fewer would have been needed. Find number used, and width of pavement.

Ans. 1320; 7 ft. 6 in.

117. A board contains 14 sq. ft.; but when length and breadth are both diminished 6 inches, its area is only $9\frac{3}{4}$ ft. What are its dimensions? *Ans.* 7 and 2 ft.

118. *A* and *B* have square allotments. Being asked their size, *A* answered, "mine is 12 yds. shorter than *B's*, and together they contain 1 r. 30 p. $2\frac{1}{2}$ yds." Find side of each. *Ans.* 26 and 38 yds.

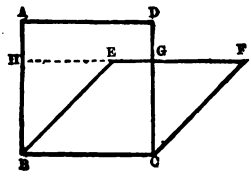
119. The size of a pane of glass is such that were its length diminished and breadth increased 3 inches, its area would 3 inches *more*; but if its length were diminished and breadth increased 6 inches, its area would be 12 inches *less*. Find area. *Ans.* 165 inches.

120. There are 2 square rooms of different sizes, such that if a side of each were to form the adjacent sides of a third room, their sum would be 33 ft., and its carpeting, at 6*s.* a yd., £9. Find cost of flooring each of the squares at the same rate. *Ans.* £10 16*s.*; £7 10*s.*

THE RHOMBUS AND RHOMBOID.

1. If the frame be removed whole from a *square* slate A B C D, and wrested out of the perpendicular at the joints, as E B C F, it will present the figure of a Rhombus. If the slate be the usual *oblong*, then the wrested frame is a Rhomboid.

Fig. 7.



2. As the sides A B and D C of the frame decline from the perpendicular, the area or space enclosed becomes less and less,

until finally A B and D C form one straight line, when of course the area vanishes altogether.

3. It is evident, therefore, that the area of a parallelogram is the product of two adjacent sides, *only when those sides are at right angles*, or perpendicular to each other.

4. It is equally evident, that the triangle G F C is similar and equal to the triangle H G B (Euclid, I. 26.), and consequently, that the area of E B C F is equal to the area of H B C G, viz. B C × H B.

5. Now H B = G C, which is the perpendicular height of E B C F, *i. e.* the perpendicular distance between two of its opposite sides. Hence the general rule for determining the area of a rhombus or rhomboid, viz., "find the product of its base and perpendicular height."

6. This rule may be otherwise expressed thus: "multiply the length of any side of a parallelogram by its perpendicular distance from the side opposite." The result is its area; and this obviously includes the square and oblong, as well as the rhombus and rhomboid.

Given, base and perpendicular height; find area.

N.B. Area = base × ⊥ height.

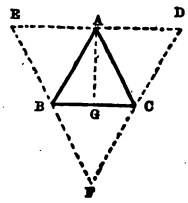
	Base.	⊥ *height.*	*Ans. Area.*
1.	18 inches.	13 inches.	1 ft. 90 inches.
2.	$8\frac{3}{4}$ yds.	$9\frac{1}{2}$ yds.	$83\frac{1}{8}$ yds.
3.	26 yds. 2 ft.	23 yds. 1 ft.	622 yds. 2 ft.
4.	93 yds. 1 ft.	106 yds. 2 ft.	9955 yds. 5 ft.
5.	23 yds. 0 ft. 2 in.	27 yds. 2 ft. 6 in.	641 yds. 6 ft. 60 in.
6.	25 yds. 2 ft. 8 in.	23 yds. 1 ft. 4 in.	606 yds. 8 ft. 80 in.
7.	7 ch. 75 lks.	13 ch. 94 lks.	10 a. 8 ch. 3500 lks.
8.	19 ch. 74 lks.	15 ch. 78 lks.	31 a. 1 ch. 4972 lks.
9.	24 p. 4 yds.	23 p. $0\frac{1}{2}$ yd.	3 a. 2 r. 10 p. $29\frac{1}{2}$ yds.
10.	69 p. $1\frac{1}{2}$ yds.	74 p. 1 yd.	32 a. 0 r. 18 p. $23\frac{1}{2}$ yds.

THE TRIANGLE.

1. Whenever a parallelogram is bisected by its diagonal, the result is two exactly equal triangles. (Euclid, I. 34.)

2. A triangle, therefore, may always be regarded as *half of a parallelogram* having the same base and perpendicular height (Euclid, I. 41.); and accordingly its area is easily computed whenever these two are known.

Fig. 8.



For, as area of ▱ = base × ⊥ height, so, area of △ = ½ (base × ⊥ height.)

3. In the above *fig.*, B C is the *base*, and A G the *height* of the △ A B C, which is half of any of the 3 ▱s A B C D, A C B E, A B F C.

Given, base and perpendicular height; find area.

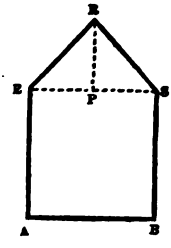
N.B. Area $\triangle = \frac{\text{base} \times \perp \text{height}}{2}$.

	Base.	⊥ *height.*	*Ans. Area.*
1.	17½ yds.	4¾ yds.	41⅛ yds.
2.	53 yds. 1 ft.	46 yds. 2 ft.	1244 yds. 4 ft.
3.	186 yds. 2 ft.	213 yds. 1 ft.	19911 yds. 1 ft.
4.	46 yds. 2 ft. 8 in.	25 yds. 2 ft. 8 in.	606 yds. 8 ft. 80 in.
5.	15 ch. 78 lks.	19 ch. 74 lks.	15 a. 5 c. 7486 lks.
6.	46 p. 1 yd.	24 p. 4 yds.	3 a. 2 r. 10 p. 29½ yds.
7.	27 ch. 88 lks.	3 ch. 87½ lks.	5 a. 4 c. 175 lks.
8.	69 p. 1½ yds.	148 p. 2 yds.	32 a. 0 r. 18 p. 23½ yds.

THE GABLE END.

1. This is merely a combination of the square or oblong and triangle, and its area, therefore, is the sum of their areas.

Fig. 9.



2. The point of junction of roof and wall, as E and S, is called the eave or spring (*i. e.* of roof). P R is the perpendicular height of the roof; and the pitch of the roof is said to be high or low, according as P R is great or small. R, the summit of the roof, is called the ridge. E S is the width or span.

Ex. Given AB = 34 ft., AE = 26 ft., and PR = 19 ft.
Here area of gable = area $\square$ $ABES$ + area $\triangle$ RES.

$$= (34 \times 26) \text{ ft.} + \frac{34 \times 19}{2}$$

$$= 884 + 323 = 1207 \text{ sq. ft.}$$

Case I.

1. Perpendicular height from floor to eave is 9 yds.; do. from eave to ridge is 23 ft.; width 32 ft. Find *total area.* *Ans.* 136 yds. 8 ft.
2. Do. 29 ft., 9 yds., 35 ft. *Ans.* 165 yds. 2 ft. 6 in.
3. Do. 26½ ft., 19 ft., 29½ ft. *Ans.* 118 yds.
4. Do. 19½ ft., 15½ ft., 23½ ft. *Ans.* 71 yds. 1 ft. 54 in.
5. From floor to ridge 67½ ft., floor to eave 13 yds., width 47 ft. *Ans.* 278 yds. 0 ft. 108 in.
6. Do. 59¾ ft., 34½ ft., 13 yds. *Ans.* 204 yds. 1 ft. 126 in.
7. From floor to ridge 57½ ft., eave to ridge 29 ft., width 37 ft. *Ans.* 176 yds., 7 ft.
8. Do. 65¾ ft., 31½ ft., 43 ft. *Ans.* 238 yds. 8 ft.

Case II.

1. A gable-end is 26 ft. wide, 23 ft. from floor to spring, and 19 ft. from spring to ridge, ($\perp$ ht.) Find cost of facing the whole with stucco at 3*s.* 4½*d.* per sq. yd. *Ans.* £15 16*s.* 10½*d.*

2. Paint 2 gable ends, each 19½ ft. wide, 10½ yds. high to eave, and 3½ yds. more to ridge, at 8½*d.* per sq. ft. *Ans.* £50 15*s.* 2½*d.*

3. Do. each 29½ ft. wide, 11¼ yds. to spring, and 2¾ yds. more to ridge, at 9¾*d.* per sq. ft. *Ans.* £90 15*s.* 7$\frac{19}{32}$*d.*

4. Gable-end is 9⅔ yds. wide, 45½ ft. from floor to ridge, and 9½ yds. from floor to spring. Find cost of rough-casting it at 1*s.* 8¼*d.* per sq. yd. *Ans.* £10 1*s.* 2¼*d.*

5. Gable-end is 15½ yds. span, 12½ yds. from floor to ridge, and 2¾ yds. from eave to ridge. What should be a contractor's estimate for painting 16 such gables at 2¼*d.* per sq. ft.? *Ans.* £222 6*s.* 6¾*d.*

6. Do. 29½ ft. span; *A* to *E* 25 ft.; *P* to *R* 19 ft. After deducting 23 yds. 5 ft. 36 in. for door and windows, find cost of painting gable at 1*s.* 9*d.* per sq. yd. *Ans.* £7 16*s.* 7½*d.*

7. Do. 26 ft. span; *A* to *E* 19 ft.; *P* to *R* 8½ ft. Only ⅓ of this area are brick. Find cost of stucco for bricks at 2*s.* 9¾*d.* per sq. yd. *Ans.* £5 8*s.* 6½*d.*

8. Do. 27½ ft. span; *B* to *S* 9 yds.; *AB* to *R* 16 yds. 1 ft. In it are 3 windows, each (7½ × 4½ ft.), and a door (7½ × 3½ ft). The wall surface of 21 such gables is to be stuccoed at 3*s.* 4½*d.* per sq. yd. Contractor *A* sends in an estimate for £300, and *B* for £400. How far is each above or below the mark? (*Vid.* "Carpet and Paper." Note 4.]

Ans. { *A*, £61 5*s.* 3¾*d.* below.
B, £38 14*s.* 8¼*d.* above.

9. A house has 2 similar and equal gable ends, each 32 ft. span; *AB* to *R* 65 ft.; *P* to *R* 29 ft. One end is faced with bricks (9 × 4) inches; the other with slates (16 × 12) inches. Bricks being 20*s.* 10*d.* per thousand, and slates 1 guinea per gross, find cost of each facing.

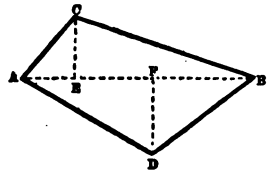
Ans. { *B*, £6 14*s.* 8*d.*
S, £8 16*s.* 9*d.*

THE TRAPEZIUM.

1. The trapezium being a 4-sided figure which is neither square nor oblong, rhombus nor rhomboid, its area can be determined only by reference to the triangle. And since

it may be divided into 2 $\triangle$s, by joining either pair of opposite angles, as A B; then, by dropping perpendiculars upon this diagonal from the other pair of angles, as C E and D F, the area is readily computed; for since the trapezium consists of the 2 $\triangle$s, A B C and A B D, it follows that

Fig. 10.



Area trapezium = area (A B C + A B D)

$$= \frac{AB \cdot CE}{2} + \frac{AB \cdot DF}{2} = \frac{AB\,(CE + DF)}{2}.$$

i. e. Area trapezium = diagonal × half sum of perpendiculars.

Ex. Let AB = 88 yds., CE 60 yds., and DF 50 yds.

$$\text{Then Area trapezium} = \frac{88 \times 60}{2} + \frac{88 \times 50}{2}$$

$$= (2640 + 2200) \text{ sq. yds.} = 1 \text{ acre.}$$

Or thus: —

$$\text{Area} = 88 \left(\frac{60 + 50}{2}\right) = (88 \times 55) \text{ yds.} = 1 \text{ acre.}$$

2. If the sides of any *trapezium* be bisected, and lines drawn joining the points of bisection, the result is a *parallelogram*, a fact which the student may impress upon his mind by repeated trials.

Case I. Given, diagonal and 2 perpendiculars upon it from the opposite angles: find area.

N.B. Area trapezium = diagonal × half sum of perpendiculars.

	Diagonal.	$\perp$ *A.*	$\perp$ *B.*
1.	35 yds.	17 yds.	19 yds.
2.	248 yds.	137 yds.	96 yds.
3.	23 yds.	27 yds.	38 yds.
4.	47 yds.	49 yds.	56 yds.
5.	39 ft.	29½ ft.	36½ ft.

	Diagonal.	⊥ *A.*	⊥ *B.*
6.	296½ ft.	347½ ft.	476½ ft.
7.	79 poles.	53 p.	67 p.
8.	2 c. 48 lks.	9 c. 60 lks.	13 c. 70 lks.
9.	23 c. 23 lks.	17 c. 17 lks.	29 c. 29 lks.
10.	39 p.	36 p. 2¾ yds.	29 p. 2¾ yds.

Answers to Case I.

630 yds.; 28892 yds.; 747½ yds.; 2467½ yds.; 143 yds.; 13573½ yds.; 29 a. 2 r. 20 p.; 2 a. 8 c. 8920 lks.; 53 a. 9 c. 6329 lks.; 8 a. 0 r. 7 p.

Case II. Given, area and 2 perpendiculars: find diagonal.

N.B. Diagonal × half sum of ⊥rs = area.

$$\therefore \text{Diagonal} = \frac{\text{area}}{\text{half sum \&c.}}$$

	Area.	⊥ *A.*	⊥ *B.*
1.	1495 sq. yds.	27 yds.	88 yds.
2.	4935 sq. yds.	98 yds.	112 yds.
3.	1260 sq. yds.	84 yds.	38 yds.
4.	43338 sq. yds.	137 yds.	96 yds.
5.	2467½ sq. yds.	28 yds.	24½ yds.
6.	3611½ sq. yds.	68½ yds.	48 yds.
7.	5 a. 7 c. 7840 lks.	19 c. 20 lks.	27 c. 40 lks.
8.	107 a. 9 c. 2658 lks.	58 c. 58 lks.	34 c. 34 lks.

Answers to Case II.

46 yds.; 47 yds.; 35 yds.; 372 yds.; 94 yds.; 62 yds.; 2 c. 48 lks.; 23 c. 23 lks.

Case III. Given, area and diagonal with one perpendicular; find the other.

N.B. Let D represent the diagonal, P, the known perpendicular, and x the unknown do.

Then, since $\left(\frac{x}{2} + \frac{P}{2}\right) D = \text{area}$,

$$\therefore \frac{D\,x}{2} = \text{area} - \frac{D\,P}{2}$$

$$\therefore x = \left(\text{area} - \frac{D\,P}{2}\right) \cdot \frac{2}{D}$$

	Area.	*Diagonal.*	*Perpendicular.*
1.	28892 sq. yds.	248 yds.	96 yds.
2.	9870 sq. yds.	47 yds.	224 yds.
3.	11 a. 5 c. 5680 lks.	2 c. 48 lks.	54 c. 80 lks.
4.	215 a. 8 c. 5316 lks.	23 c. 23 lks.	117 c. 16 lks.

Answers to Case III.

137 yds.; 196 yds.; 38 c. 40 lks.; 68 c. 68 lks.

THE TRAPEZOID.

Fig. 11.

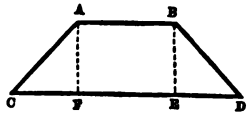


Fig. 12.

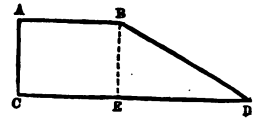
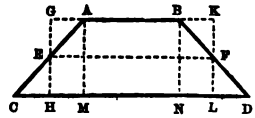


Fig. 13.



1. When the trapezium has 2 sides parallel, it is called a trapezoid; and its area is the *product of its mean breadth and perpendicular height.*

2. The *mean breadth* = ½ sum of parallel sides; the word "mean," in such cases as this, signifying "medium," or "average."

3. The method of determining a *triangular area* may be deduced and illustrated by reference to this figure. For if, while C D remains the same, A B be diminished to a *point*, the trapezoid becomes a triangle, whose "mean breadth" is $\frac{A B + C D}{2}$, *i. e.* $\frac{0 + C D}{2}$ or $\frac{C D}{2}$. And its area therefore = $\frac{C D}{2} \times B E$, *i. e.* ½ (base × height), as before.

4. "Area trapezoid = mean breadth × ⊥ ht." This may be shown as follows:

Take E and F, the middle points of A C and B D (*fig.* 13.), and join these points of bisection. Then E F is the *mean breadth* of the trapezoid.

Through E and F draw G H and K L, perpendicular to C D and A B produced. Also draw A M and B N parallel to G H and K L.

Then (Euc. I. 26.), $\triangle$ E C H = $\triangle$ E A G;

and C H = G A = H M;

$\therefore$ C H is half of C M.

Similarly, L D is half of N D;

$\therefore$ C H + L D = $\frac{1}{2}$ (C M + N D).

Take away (C H + L D) from C D, and there remains H L, *i. e.* (M N + H M + N L), which is exactly the same as (A B + C H + L D).

But H L + (A B + C H + L D) = A B + C D, $\therefore$ H L or E F = $\frac{\text{A B + C D}}{2}$ = mean breadth.

And since trapezoid A B C D = rectangle G H L K, the area of which = H L × H G = E F × A M, $\therefore$ area trapezoid = mean breadth × ⊥ height. Q.E.D.

Ex. Let *AB* and *CD* (*Figs.* 11, 12.) be the parallel sides, and *BE* the perpendicular height; *AB* being 10 yds., *CD* 20 yds., and *BE* 8 yds.; then,

Area *ABCD* = 15 × 8 = 120 sq. yds.

This result may be proved correct thus:

Fig. 11. Here F C = E D = 5 yds. Then (Euc. I. 26.)

$$\triangle \text{A F C} = \triangle \text{B E D} = \frac{\text{B E} \times \text{E D}}{2} = 20 \text{ sq. yds.}$$

$$\square \text{A B F E} = \text{A B} \times \text{B E} = 10 \times 8 = 80 \text{ sq. yds.}$$

$\therefore$ Area A B C D = 80 + 20 + 20 = 120 sq. yds.

Fig. 12. Here C E = E D = 10 yds.

$$\therefore \triangle \text{B E D} = \frac{10 \times 8}{2} = 40 \text{ sq. yds.}$$

and $\square$ A B C E = 10 × 8 = 80 sq. yds.

$\therefore$ Area A B C D = 80 + 40 = 120 sq. yds.

Case I. Given, 2 parallel sides and perpendicular distance between them: find area.

N. B. Area trapezoid = perpendicular × half sum of parallels.

	Perpendicular.	*∥A.*	*∥B.*
1.	4 ft. 8 in.	1 ft. 5 in.	1 ft. 11 in.
2.	19 ft.	37 ft.	25 ft.
3.	62 yds.	19½ yds.	26½ yds.
4.	158 p.	26½ p.	33½ p.
5.	4 c. 96 lks.	27 c. 04 lks.	19 c. 02 lks.
6.	69 c. 69 lks.	51 c. 51 lks.	87 c. 87 lks.

Answers to Case I.

7 ft. 112 in.; 65 yds. 4 ft.; 1426 yds.; 29 a. 2 r. 20 p.; 11 a. 4 c. 2288 lks.; 485 a. 6 c. 6961 lks.

Case II. Given, area and 2 parallels: find perpendicular.

N. B. Perpendicular × half sum of ∥s = area.

$$\therefore \text{Perpendicular} = \frac{\text{Area}}{\text{half sum \&c.}}$$

	Area.	*∥A.*	*∥B.*
1.	2990 yds.	76 yds.	54 yds.
2.	21669 yds.	68½ yds.	48 yds.
3.	3611½ yds.	96 yds.	137 yds.
4.	29 a. 2 r. 20 p.	67 p.	53 p.
5.	26 a. 0 c. 0280 lks.	28 c. 80 lks.	41 c. 10 lks.
6.	2 yds. 2 ft. 110 in.	4 ft. 6 in.	6 ft. 4 in.

Answers to Case II.

46 yds.; 372 yds.; 31 yds.; 79 p.; 7 c. 44 lks.; 3 ft. 10 in.

Case III. Given, area and perpendicular height, with one parallel side: find the other.

N. B. Let A represent the *known* parallel, x the *unknown* do., and P the perpendicular.

Then, since $\left(\frac{x}{2}+\frac{A}{2}\right) P = \text{area}$,

$$\therefore \quad \frac{P\,x}{2} = \text{area} - \frac{P\,A}{2}$$

$$\therefore \quad x = \left(\text{Area} - \frac{P\,A}{2}\right) \cdot \frac{2}{P}$$

	Area.	*∥A.*	*Perpendicular.*
1.	28892 yds.	96 yds.	248 yds.
2.	143 yds.	29½ ft.	13 yds.
3.	2 a. 8 c. 8920 lks.	13 c. 70 lks.	2 c. 48 lks.
4.	13 a. 3 r. 23 p.	36½ p.	39 p.

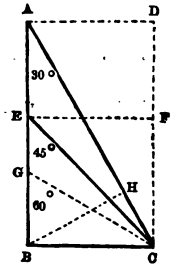
Answers to Case III.

137 yds.; 36½ ft.; 9 c. 60 lks.; 77½ p.

THE RIGHT-ANGLED TRIANGLE.

1. It has been pointed out, that a triangle may always be considered as half of a parallelogram. And if the bisected □ be rectangular, as A B C D, (*vid.* "Square," Note 3.), it is evident that each of its halves must be a rt. ∠d. △, as A B C, which has its rt. ∠ at B; the side A C, which subtends or is opposite to the rt. ∠, being called the hypothenuse.

Fig. 14.



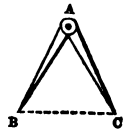
2. Since every square or oblong has 4 rt. ∠s, (or "upright ∠s," being formed by lines upright or perpendicular to each other), each of which is called an angle of 90 degrees, (usually written 90°); and since a rhombus and rhomboid are only a square and oblong twisted awry, so that while 2 of its angles become less, the other 2 become proportionally greater, and the *sum* of the

4 ∠s in each always = 4 rt. ∠s, or 360°; it follows, that the *sum* of the 3 ∠s in any Δ whatsoever always = 2 rt. ∠s, or 180°. (Euclid, I. 32. and 34., II. 9.)

3. When the perpendicular sides, *i. e.* the sides containing the rt. ∠, are equal, as E B and B C, the Δ is *isosceles*, being ½ a square; when unequal, as A B and B C, the Δ is *scalene*, being ½ an oblong.

4. Every Δ A B C may be regarded as a pair of compasses, extended for measurement; its legs being equal or unequal at pleasure, and the joint or hinge at either A, B, or C.

Fig. 15.



[Here B C is the side which "*subtends*," or *stretches under*, the ∠ B A C.]

5. Now since B C, the base or line measured, must be longer or shorter in proportion to the width of the compass opening, *i. e.* according as ∠ B A C is greater or less; and since, on the other hand, the compasses must be opened more or less widely, *i. e.* the ∠ B A C must be greater or less, according as B C is longer or shorter; there follows that general rule with regard to triangles, so important to be remembered by every student of Euclid or geometry, viz., *sides depend upon opposite angles, and angles depend upon opposite sides;* so that either being known, the magnitude of the other may be judged of, if not exactly estimated. (*Vid.* Euclid, Bk. I., Props. 4—12. 16. 18—21., 23—26, 33, 34., 47, 48.)

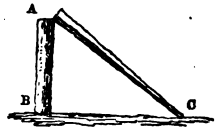
6. The accurate determination of these relative magnitudes is the special province of TRIGONOMETRY. But it is easy to see and remember that in any rt. ∠d Δ E B C (*vid. fig.*), whenever side E B = side B C, then ∠ B C E = ∠ B E C (Note 4.); and that, since the sum of the 3 ∠s of any Δ = 2 rt. ∠s or 180° (Note 2.), and since ∠ E B C = 90°, being a rt. ∠, the *sum* of the other 2 ∠s must = 90° also, and ∴ each of them = ½ a rt. ∠, or 45°.

7. Similarly, whenever one of the 2 acute ∠s, in a rt. ∠d Δ = 45°, it is known at once that the other must = 45° too, and consequently that their "opposite sides" (Note

5.), *i. e.* the sides containing the rt. ∠, are equal to each other.

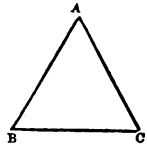
e. g. If a ship's mast B A C, being struck by shot or snapped in a gale, break off at A, so that the upper portion A C, falling to the deck, makes with it an angle of 45° at a distance of 36 ft. from the foot of the mast, then it is known that A B must = 36 ft. too; and from these data the length of A C is determined to be 50·91 ft. (Euclid, I. 47.), so that the original height of the mast was 87 ft. nearly.

Fig. 16.



8. If the triangle be *equilateral*, it is also *equiangular* and *vice versâ*, (Euclid, I. 5.); and each angle therefore = $\frac{1}{3}$ of 180° = 60° (Note 2.) Now in the △ A B C, (*fig.* 17.) if one side, as B C, be measured and found 40 yds., and the adjacent ∠s at B and C each = 60°, then the 3rd ∠ at A = 60° also, and each of the sides B A and A C = 40 yds. (Note 5.)

Fig. 17.



9. Again: if any 2 sides in the above △, as A B and B C, be found equal, each being 40 yds., and the enclosed ∠ A B C = 60°, then the other 2 ∠s at A and C each = 60°, and A C = 40 yds. (*Vid.* Appendix.)

10. It will be worth remembering too, that if an observer, after walking 100 yds. from B to G, (*fig.* 14.) finds the ∠ B G C = 60°, he will have to walk just twice the distance farther before he finds the ∠ B A C = 30°; *i. e.* B A will = 3 B G, whenever the angles B G C and B A C = 60° and 30° respectively.

a. It is proved by Euclid (Bk. I., Prop. 47.) that the square described upon the hypothenuse A C of any rt. ∠d. △ A B C contains as much space as the 2 squares upon the sides A B and B C which contain the rt. ∠; *i. e.* in brief, $A C^2 = A B^2 + B C^2$. [And this can be shown very simply by means of squares accurately cut out of paper, &c.]

β. Any 2 of these being given, the 3rd can be foun
For $AB^2 + BC^2 = AC^2$; $\therefore AB^2 = AC^2 - BC^2$, and BC^2
$AC^2 - AB^2$; and $\therefore AC = \sqrt{AB^2 + BC^2}$, &c.

γ. This may be readily proved in the case of a Δ whc
sides are either the numbers 3, 4, 5, or *any equimultiples them*, such as 6, 8, 10; 15, 20, 25, &c.; since any 2 these being given, the 3rd will be found as an *integer* complete number. If the two given sides be in any oth proportion, the 3rd can only be found *approximately*, l decimals.

δ. Whenever the sides are in proportion of 3, 4, 5, the is at once known to be rt. $\angle$d.

Case I. Given, the 2 $\perp$ sides, to find hypothenuse. (*Vi fig.* 14. and notes β, γ.

	AB.	*BC.*	*Ans. AC.*
1.	9	12	15
2.	87	116	145
3.	108	81	135
4.	332	249	415
5.	75	95	121·04
6.	89	65	110·2
7.	236	177	295
8.	65	38	75·29

Case II. Given, hypothenuse and one $\perp$ side: find th other. (*Vid.* note β.)

	AB.	*AC*	*Ans. BC.*
1.	148	185	111
2.	396	495	297
3.	176	298	240·47
4.	189	467	427·04
5.	178	367	320·94
6.	237	395	316

Case III. Given, 2 $\perp$ sides: find area of Δ.

[Since $\Delta ABC = \frac{1}{2} \square ABCD$, whose area $= AB \times BC$, area $ABC = \frac{1}{2}(AB \times BC)$.]

	AB.	*BC.*	*Ans. Area.*
1.	16 in.	12 in.	$\frac{3}{4}$ sq. ft.
2.	59 ft.	77 ft.	$252\frac{7}{18}$ sq. yds.
3.	$116\frac{1}{2}$ yds.	$87\frac{1}{2}$ yds.	$5096\frac{7}{8}$ yds.
4.	$27\frac{1}{2}$ poles	39 p.	3 a. 1 r. $16\frac{1}{4}$ p.
5.	13 c. 65 lks.	9 c. 56 lks.	6 a. 5 c. 2470 lks.
6.	23 p.	$37\frac{1}{2}$ p.	2 a. 2 r. $31\frac{1}{4}$ p.
7.	$37\frac{1}{2}$ yds.	$47\frac{1}{2}$ yds.	890 yds. 5 ft. 90 in.
8.	19 c. 12 lks.	27 c. 30 lks.	26 a. 0 c. 9880 lks.

Case IV. Given, hypothenuse and one $\perp$ side: find area Δ.

[Here find 2nd $\perp$ by Case II., and then proceed as in Case III.]

	AB.	*AC.*	*Ans. Area.*
1.	15 in.	25 in.	1 ft. 6 in.
2.	76 ft.	95 ft.	$240\frac{2}{3}$ yds.
3.	$124\frac{1}{2}$ ft.	$207\frac{1}{2}$ ft.	1148 yds. 1 ft. 6 in.
4.	25 p.	$37\frac{1}{2}$ p.	2 a. 0 r. $28\frac{3}{4}$ p.
5.	4 c. 11 lks.	6 c. 85 lks.	1 a. 1 c. 2614 lks.
6.	9 c. 85 lks.	13 c. 79 lks.	4 a. 7 c. 4770 lks.
7.	$104\frac{1}{4}$ p.	$173\frac{3}{4}$ p.	45 a. 1 r. $5\frac{3}{8}$ p.
8.	13 c. 07 lks.	17 c. 70 lks.	7 a. 7 c. 9625 lks.

Case V. Given, area and one $\perp$ side: find 2nd side and hypothenuse.

[Let a represent the known $\perp$, and x the unknown do. Then, since $\frac{a\,x}{2} = \text{area}$, $\therefore x = \frac{2 \text{ area}}{a}$ &c.]

	Area.	*$\perp$ side.*	*Ans. $\perp$ and hyp.*
1.	294 yds.	28 yds.	21 and 35 yds.
2.	1 ft. 6 in.	15 in.	20 and 25 in.
3.	$\frac{3}{4}$ ft.	16 in.	12 and 20 in.
4.	252 yds. 3 ft. 72 in.	25 yds. 2 ft.	59 and 97 in.
5.	6 a. 2 r. $32\frac{1}{2}$ p.	55 p.	39 and 67·424 p.
6.	5 a. 1 r. $22\frac{1}{2}$ p.	$37\frac{1}{2}$ p.	46 and 59·35 p.
7.	3 a. 0 c. 7428 lks.	8 c. 22 lks.	6 c. 85 lks. and 10 c. 70 l.
8.	13 a. 0 c. 4940 lks.	19 c. 12 lks.	13 c. 65 l. and 23 c. 49 l.

Case VI. Given, hypothenuse only: find area, and ⊥ sides, when they are equal. (*Vid.* Notes 3. and 6.)

[Let x and y be the 2 ⊥s.

Now hyp.$^2 = x^2 + y^2 = 2x^2$ or $2y^2$, by hypothesis.

$$\therefore x \text{ and } y \text{ each } = \sqrt{\frac{\text{hyp}^2.}{2}}, \text{ \&c.}$$

N.B. Where the hypothenuse is not a whole number, take as its root the whole number nearest to the root found.]

	Hypothenuse.	*Ans. 2 sides.*	*Area.*
1.	$6\frac{1}{4}$ ft.	53·033 in.	1 yd. 0 ft. $110\frac{1}{2}$ in.
2.	$22\frac{1}{2}$ ft.	15·9 p.	3 r. $6\frac{9}{16}$ p.
3.	29 c. 95 lks.	21 c. 18 lks.	22 a. 4 c. 2962 lks.
4.	8·485 in.	6 in.	18 in.
5.	22·625 in.	16 in.	$\frac{9}{8}$ ft.
6.	35·3553 ft.	25 ft.	34 yds. 6 ft. 72 in.
7.	33 c. 16 lks.	23 c. 45 lks.	27 a. 4 c. 9513 lks.

Case VII. Given, hyp. only: find area, and ⊥ sides, when not equal, but in proportion of 3 : 4.

[Since the 2 ⊥s are in the proportion 3 : 4, the *sum* of their lengths must proportionally be (3+4) or 7. Now let x represent the actual length of this *sum;* then the respective lengths of the 2 ⊥s will be $\frac{4}{7}x$ and $\frac{3}{7}x$. And since hyp.^{2} $= \frac{16}{49}x^2 + \frac{9}{49}x^2 = \frac{25}{49}x^2$, $\therefore x = \frac{7}{5}$ hyp. &c. And any other proportion may be dealt with in the same way.]

	Hypothenuse.	*Ans. 2 sides.*	*Area.*
1.	95 yds.	57 and 76 yds.	2166 yds.
2.	115 yds.	69 and 92 yds.	3174 yds.
3.	8 yds. 0 ft. 7 in.	$19\frac{3}{5}$ and $14\frac{3}{4}$ ft.	16 yds. 1 ft. 6 in.
4.	2 fur. $13\frac{3}{4}$ p.	$56\frac{1}{4}$ and 75 p.	13 a. 0 r. $29\frac{3}{8}$ p.
5.	14 c. 90 lks.	8 c. 94 lks. and 11 c. 92 lks.	5 a. 3 c. 2824 lks.
6.	37 yds. 2 ft. 9 in.	$68\frac{1}{4}$ and 91 ft.	345 yds. 0 ft. 54 in.

Case VIII. Given, 2 ⊥ s, A B and B C: find B H, the ⊥ upon hyp. from rt. ∠. (*Vid. fig.* 14., and Euclid, II. 13.)

[Here B H divides A C into two portions, A H and H C. Let x represent the length of A H; then H C = A C − x.

Now $BH^2 = AB^2 - AH^2$, and also $= BC^2 - CH^2$; $\therefore AB^2 - AH^2 = BC^2 - CH^2$, or $AB^2 - x^2 = BC^2 - (AC - x)^2$, from which equation the value of x, and therefore of BH, is readily determined.]

	AB.	*BC.*	*Ans.* *BH.*
1.	57 yds.	76 yds.	45½ yds. nearly.
2.	92 yds.	69 yds.	55·2 yds. „
3.	4 ft. 3 in.	5 ft. 8 in.	40·8 in. „
4.	88 ft.	22 yds.	52·8 ft. „
5.	40½ p.	54 p.	32·4 p. „
6.	93 c. 80 lks.	70 c. 35 lks.	68 ch. „

[Obs. Base *AC* is not cut proportionally by *BH*, *i. e.* *AH* is not to *HC* as *AB* : *BC*, unless $\angle ABH = \angle CBH$. *Vid.* Euclid, VI. 3.]

Case IX. Given, hypothenuse and *sum* of 2 ⊥s, find ⊥s and area.

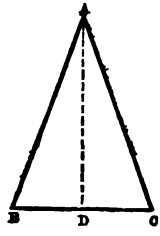
Ex. Suppose hypothenuse = 5 and sum of perpendiculars = 7.
Let one perpendicular = x, then the other = $7 - x$.
Now $5^2 = x^2 + (7-x)^2 = 2x^2 - 14x + 49$.
From which quadratic, $x = 4$ or 3, and $\therefore 7 - x = 3$ or 4, &c.

	Hyp.	*Sum* ⊥s.	*Ans.* *Two* ⊥s.	*Area.*
1.	25 yds.	35 yds.	20 & 15 yds.	150 yds.
2.	20 yds.	25 yds.	19·114 & 5·886 yds.	57 yds.
3.	31 yds. 2 ft.	44 yd. 1 ft.	76 ft. & 19 yds.	240 yds. 6 ft.
4.	4 ft. 2 in.	5 ft. 10 in.	2 ft. 6 in. & 3 ft. 4 in.	4 ft. 24 in.
5.	35 p.	49½ p.	25 & 24½ p.	1 a. 3 r. 26¼ p.
6.	37 c. 80 lk.	52 c. 92 lk.	30 c. 24 lk. & 22 c. 68 lk.	34 a. 2 c. 9216 lk.

THE ISOSCELES TRIANGLE.

1. The isosceles △ (*ἰσος*, equal, *σκελος*, leg) having two legs or sides equal, and therefore 2 ∠s equal (*vid.* Rt. ∠d △, Note 5,) is ½ a square when it contains a rt. ∠, and ½ a rhombus when it is either acute or obtuse ∠d. (*Vid.* Rt. ∠d △, Note 3. and Euclid's definitions.)

Fig. 18.



N.B. The student will see this clearly if he draws a square and rhombus, and then joins the opposite ∠s of each. The two *diagonals* of the rhombus will form

the base of an acute and obtuse ∠d Δ respectively. (*Vid. fig.* in "the Diamond," *infrd.*

2. When the ∠ enclosed between the equal sides is an ∠ of 90°, *i. e.* a rt. ∠ (which is the case when the Δ is ½ a square), its area, &c., may be found as a rt. ∠d Δ.

3. When the included ∠ contains either more or less than 90°, *i. e.* when the equal sides are not at rt. ∠s to each other, (which is the case when the Δ is ½ a rhombus,) then the ⊥ height of the Δ must be found, since area Δ = ½ (base × ⊥ height); and the simplest mode of doing this is to find the length of A D, (*vid. fig.*) for the following reason.

4. The ⊥ A D, being drawn from A to meet the base B C, *bisects* it; *i. e.* B D = D C = ½ B C. A D consequently is at once found from the equation $A D = \sqrt{A B^2 - B D^2}$ or $\sqrt{A C^2 - D C^2}$. (*Vid.* Rt. ∠d Δ, Note *a*.)

5. The ∠ B A C also is bisected by A D. And when A D = B D, then of course ∠ D B A = ∠ D A B, *i. e.* each = 45° or ½ rt. ∠.

Case I. Given, base and ⊥ height: find area and side. (Vid. Notes 3 and 4.)

	Base.	⊥ *ht.*	*Ans. Area and Side.*	
1.	16 in.	13 in.	104 in.	15·264 in.
2.	37½ p.	59 p.	6 a. 3 r. 26½ p.	47·712 p.
3.	15 c. 78 lks.	19 c. 74 lks.	15 a. 5 c. 7486 lks.	21 c. 26 lks.
4.	53 yds. 1 ft.	46 yds. 2 ft.	1 r. 1 p. 15 yds. 4 ft.	161·245 ft.

Case II. Given, base and side: find area and ⊥ height. (Vid. Notes 3 and 4.)

	Base.	*Side.*	*Ans. Area and* ⊥ *ht.*	
1.	3 ft. 4 in.	2 ft. 1 in.	2 ft. 12 in.	15 in.
2.	77 yds. 1 ft.	48 yds. 1 ft.	1121 yds. 3 ft.	29 yds.
3.	59 p.	95·425 p.	16 a. 2 r. 37½ p.	90¾ p.
4.	7 c. 89 lks.	12 c. 48 lks.	3 a. 8 c. 9372 lks.	9 c. 87 lks.

Case III. Given, side and ⊥ height: find base and area.

[*N. B.* $B D = \sqrt{A B^2 - A D^2}$, ∴ $B C = 2\sqrt{A B^2 - A D^2}$].

	Side.	⊥ *ht.*	*Ans.*	*Base and Area.*
1.	7 ft. 11 in.	4 ft. 9 in.	12 ft. 8 in.	3 yds. 3 ft. 12 in.
2.	96 yds. 2 ft.	58 yds.	154 yds. 2 ft.	4485 yds. 3 ft.
3.	25 c. 28 lks.	19 c. 74 lks.	31 c. 58 lks.	31 a. 1 c. 6976 lks.
4.	67½ p.	40½ p.	108 p.	13 a. 2 r. 27 p.

Case IV. Given, area and ⊥ height: find base and side.

[*N. B.* $\frac{\text{B C} \times \text{A D}}{2} = \text{area}$; $\therefore$ $\text{B C} = \frac{2 \text{ area}}{\text{A D}}$, &c.]

	Area.	⊥ *ht.*	*Ans.*	*Base and Side.*
1.	2028 yds.	39 yds.	104 yds.	65 yds.
2.	901⅓ yds.	26 yds.	69 yds. 1 ft.	43 yds. 1 ft.
3.	15 a. 5 c. 7486 lks.	19 c. 74 lks.	15 c. 79 lks.	21 c. 25 lks.
4.	3 a. 2 r. 10 p. 29½ yds.	24 p. 4 yds.	46 p. 2 yds.	33 p. 9 yds.

Case V. Given, area and base: find ⊥ ht. and side. (*Vid.* Case IV.)

	Area.	*Base.*	*Ans.*	*Height and Side.*
1.	2352 yds.	112 yds.	42 yds.	70 yds.
2.	1244 yds. 4 ft.	53 yds. 1 ft.	46⅔ yds.	161·245 ft.
3.	5 a. 4 c. 0175 lks.	27 c. 88 lks.	3 c. 89 lks.	14 c. 50 lks.
4.	261⅓ yds.	37⅓ yds.	14 yds.	23⅓ yds.

Case VI. Given, area and side: find base and ⊥ ht., which are in proportion as 8 : 3.

[*N. B.* Base : ht. :: 8 : 3; $\therefore$ ½ base : ht. :: 4 : 3. Then proceed as in Rt. ∠d △, Case VII.]

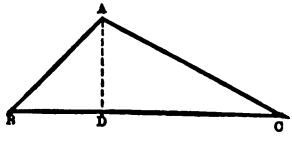
	Area.	*Side.*	*Ans.*	*Base and Height.*
1.	588 yds.	35 yds.	56 yds.	21 yds.
2.	1045⅓ yds.	46 yds. 2 ft.	74⅔ yds.	28 yds.
3.	21 a. 6 c. 0700 lks.	28 c. 94 lks.	55 c. 76 lks.	7 c. 75 lks.
4.	2 a. 1 r. 3 p.	27½ p.	33 p.	22 p.

Case VII. Given base only, when base : height :: 2 : 1; find area and sides. (*Vid.* Note 5.)

	Base.	*Ans.*	*Area.*	*Side.*
1.	76 yds.		1444 yds.	53·7 yds.
2.	75 in.		1 yd. 0 ft. 66½ in.	53·033 in.
3.	98 yds.		2401 yds.	69·3 yds.
4.	33 c. 16 lks.		27 a. 4 c. 8964 lks.	23 c. 45 lks.

THE EQUILATERAL AND SCALENE TRIANGLE.

Fig. 19.



1. The scalene triangle is $\frac{1}{2}$ trapezium, having no 2 sides, and $\therefore$ no 2 $\angle$s, equal. For equilat. $\triangle$, vid. Rt. $\angle$d $\triangle$, Note 8. and Isosc. $\triangle$, Note 4.

2. The method of solution here adopted is the same as in the Rt. $\angle$d $\triangle$, Case VIII. the grand point being to determine the $\perp$ ht. of $\triangle$, *i. e.* the length of line drawn from any convenient $\angle$, $\perp$ to the base or side opposite.

3. Trigonometry, however, indicates the following formula, which may be used to verify results obtained by the foregoing process; viz. "area of any $\triangle = \sqrt{s(s-a)(s-b)(s-c)}$;" where a, b, c, are sides of the $\triangle$, and $s = \frac{1}{2}(a + b + c)$.

4. When A B C is equilat. B D = D C = $\frac{1}{2}$ A B; $\therefore$ B D^2 = $\frac{1}{4}$ A B^2, and $\therefore$ A D = $\sqrt{\text{A B}^2 - \frac{1}{4}\text{A B}^2} = \sqrt{\frac{3}{4}\text{A B}^2}$.

Case I. Given, 3 sides of any triangle: find its area.

	The 3 Sides.	*Ans. Area.*
1.	120 yds.; 120 yds.; 120 yds.	6235 yds. nearly.
2.	60 ft.; 100 ft.; 60 ft.	1660 yds. nearly.
3.	5 ft. 9 in.; 9 ft. 7 in.; 7 ft. 8 in.	2 yds. 4 ft. 6 in. nearly.
4.	58 yds.; 64 yds.; 76 yds.	1807 yds. nearly.
5.	24 p.; 36 p.; 48 p.	2 a. 2 r. 18¼ p. nearly.
6.	9 c. 87 lks.; 8 c. 76 lks.; 7 c. 65 lks.	3 a. 2 c. 1492 lks. nearly.
7.	15 in.; 20 in.; 15 in.	111·803 in.
8.	380 yds.; 420 yds.; 765 yds.	9 a. 0 r. 38 p.
9.	13 ft.; 14 ft.; 15 ft.	9¼ yds.
10.	50 c. 25 lks.; 49 c.; 25 c. 69 lks.	61 a. 1 r. 39·68 p.

Case II. Given, height of an equil. $\triangle$: find area. (*Vid.* Note 4.)

14 yds.; 21 yds.; 16 ft.; 20 in.; 35 poles; 3 c. 84 lks.

Answers.

113·155 yds.; 254·598 yds.; 16·42 yds.; 230·94 in.; 4 a. 1 r. 27½ p.; 8 c. 5134 lks.

Case III. Given, *area* of equilat. Δ: find *side*.

[*N.B.* Let a represent area, b side or base, and p the perpendicular height.

$$\text{Now since } \frac{b \times p}{2} = a, \therefore p = \frac{2a}{b}.$$

$$\text{But } p = \sqrt{\tfrac{3}{4} b^2} \text{ (Note 4.)}; \therefore \sqrt{\tfrac{3}{4} b^2} = \frac{2a}{b}$$

$$\therefore \tfrac{3}{4} b^2 = \frac{4a^2}{b^2} \therefore b^4 = \frac{16a^2}{3} \therefore b = \sqrt[4]{\frac{16a^2}{3}}.$$

The young student need not be alarmed at this biquadratic; for b is found very easily by merely extracting the *square* root of $\frac{16a^2}{3}$ *twice in succession.*

Ex. 6235 sq. yds.; 1559 yds.; 156·317 yds.; 389 yds.

Ans. 120 yds.; 60 yds.; 19 yds.; 30 yds. (*Vid.* Reg. Pol. Note 8.)

To find the Area of a Square or Oblong from its Diagonal.

In *fig.* 14. of the Rt. $\angle$d Δ, E C and A C are diagonals of the square E B C F and oblong A B C D. Now from Euclid, I. 47., we have learned the relation existing between these *diagonals* and the *sides* of the figures which they bisect. (Euclid, I. 34.) The former therefore being known, the latter are readily discovered, and the area of the $\square$ computed accordingly.

e. g. Given, diagonal of square = 16 yds. Find sides and area.

Here, $\text{E B}^2 + \text{B C}^2$, *i. e.* $2\,\text{B C}^2 = \text{E C}^2 = 256$ sq. yds. (*Fig.* 14. above.)

$$\therefore \text{B C}^2 = \frac{256}{2} = 128; \therefore \text{B C} = \sqrt{128} = 11{\cdot}3 \text{ yds.}$$

And area square = $\text{B C}^2 = 128$ yds.

Case I. Given, diagonal only of square: find area and

	Diagonal.	*Ans. Area, and Side.*	
1.	98 yds.	4802 yds.	69·3 yds.
2.	76 yds.	2888 yds.	53·7 yds.
3.	6 ft. 3 in.	2 yds. 0 ft. 133 in.	53·033 in.
4.	$22\frac{1}{2}$ p.	3 a. 0 r. $26\frac{1}{4}$ p.	15·9 p.
5.	29 c. 95 lks.	44 a. 8 c. 5013 lks.	21 c. 18 lks.
6.	33 c. 16 lks.	54 a. 9 c. 7928 lks.	23 c. 45 lks.
7.	35·3555 ft.	69 yds. 4 ft.	25 ft.
8.	22·6255 in.	1 ft. 112 in.	16 in.

9. Diagonal of sq. room is 23 ft. Find cost of carpet 24 inch wi 3*s.* 6*d.* a yd. *Ans.* £7 14s

10. Diagonal of sq. room is 24 ft. Find cost of carpet 45 inch at 7*s.* 6*d.* a yd. *Ans.* £

11. Diagonal of square field is 847 yds. Find value at 10*s.* sq. rod. *Ans.* 5978 8

12. Find cost of fencing this field all round at 6*s.* $2\frac{1}{4}$*d.* a rod. *Ans.* £134 15

Case II. Given, diagonal and side of oblong: find are

	Diagonal.	*Side.*	*Ans. Area.*
1.	$29\frac{3}{4}$ in.	$13\frac{1}{2}$ in.	2 ft. $69\frac{3}{4}$ in.
2.	$19\frac{5}{8}$ ft.	$17\frac{1}{4}$ ft.	17 yds. 8 ft. $61\frac{3}{4}$ in.
3.	$37\frac{1}{4}$ in.	$17\frac{1}{2}$ in.	3 ft. $143\frac{1}{2}$ in.
4.	$47\frac{7}{8}$ yds.	$13\frac{3}{4}$ yd.	627 yds. 0 ft. $85\frac{1}{4}$ in.
5.	$19\frac{1}{2}$ p.	$7\frac{1}{2}$ p.	135 p.
6.	19 c. 85 lks.	5 c. 78 lks.	10 a. 9 c. 7564 lks.
7.	$47\frac{1}{4}$ p.	$25\frac{3}{4}$ p.	6 a. 1 r. $20\frac{1}{2}$ p.
8.	29 c. 78 lks.	13 c. 64 lks.	36 a. 1 c. 0508 lks.

9. Diagonal of the lid of an oblong box measures $3\frac{5}{16}$ ft., and on is 32·886 inches. Find cost of oilskin top at 4*s.* 6*d.* a sq. yd. *Ans.* 2*s.* n

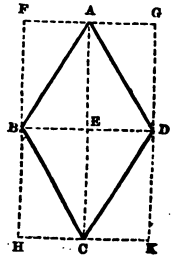
10. A rectangular field has a diagonal drain $107\frac{1}{4}$ yds. long, an only side that can be measured is 99 yds. Find area.

Ans. 3 roods,

THE DIAMOND.

1. The diamond is a rhombus, set vertically upright in general, as in the case of window panes and hatchments; and may be considered as 2 exactly equal Δs on opposite sides of the same base which forms one diagonal of the $\square$.

Fig. 20.



2. The two diagonals, A C and B D, bisect each other in the centre of the figure.

3. The dotted lines in the figure, drawn through the $\angle$s and // to the diagonals, complete the oblong, of which the diamond is exactly one half. And since area of oblong F H K G = F H × F G = A C × B D, it follows that area of diamond = $\frac{1}{2}$(A C × B D).

4. When one *side* can be measured, and also the $\perp$ upon it from the $\angle$ opposite, the area, &c. can be found as before. (*Vid.* Rhombus.)

5. When one *side* can be measured, and *only one diagonal* (suppose A C), then the other is found from the equation $BE = \sqrt{AB^2 - AE^2}$.

Case I. Given, the 2 diagonals: find area and side. (*Vid.* Note 3.)

	Diagonals.		*Ans. Area and Side.*	
1.	12 in.	16 in.	96 in.	10 in.
2.	26 yds.	34$\frac{2}{3}$ yds.	450$\frac{2}{3}$ yds.	21·2 yds.
3.	11 ft.	9 ft.	49$\frac{1}{2}$ ft.	7·1063 ft.
4.	9$\frac{3}{4}$ ft.	7$\frac{1}{2}$ ft.	36·5625 ft.	6·1504 ft.
5.	21$\frac{3}{4}$ p.	29 p.	1 a. 3 r. 35$\frac{5}{8}$ p.	18$\frac{1}{8}$ p.
6.	18 c. 92 lks.	14 c. 19 lks.	13 a. 4 c. 2374 lks.	11 c. 82$\frac{1}{2}$ lks.

Case II. Given, 1 diagonal and a side: find area and 2nd diag. (*Vid.* Note 5.)

	Diagonal.	*Side.*	*Ans., Area, and 2nd Diagonal.*	
1.	56 in.	35 in.	8 ft. 24 in.	$3\frac{1}{2}$ ft.
2.	$50\frac{2}{3}$ yds.	$31\frac{2}{3}$ yds.	2888 ft.	38 yds.
3.	$15\frac{1}{3}$ ft.	$19\frac{7}{12}$ ft.	88 ft. 24 in.	$11\frac{1}{2}$ ft.
4.	156 yds.	78 yds.	Impossible.	Why?
5.	$133\frac{1}{2}$ p.	$111\frac{1}{4}$ p.	74 a. 1 r. $1\frac{1}{2}$ p.	178 p.
6.	62 c. 96 lks.	39 c. 35 lks.	148 a. 6 c. 49 lks.	47 c. 22 lks.

Case III. Given, vertical $\angle = 60°$, and *shorter* diagonal: find area.

N. B. When vertical $\angle$ B A D or B C D $= 60°$, each of the Δs A B D, C B D is equiangular and equilateral. (Euclid, I. 34.) *Vid.* Rt. $\angle$d Δ, Notes 5—9., and Isosceles Δ, Note 4.)

The following are the *shorter* diagonals of diamonds whose vertical $\angle$ contains 60°. Find area in each instance.

16 ft.; 48 in.; 80 yds.; 72 p.; 4 c. 32 lks.; 9 ft. 4 in.

Ans. 221·696 ft.; 13 ft. 123·312 in.; 5542·56 yds.; 28 a. 0 r. 9·416 p.; 1 a. 6 c. 1621 lks.; 8 yds. 3 ft. 63·328 in.

Obs. The question might be otherwise put, as, "given, the 2 vertical $\angle$s $= 120°$;" or "obtuse $\angle = 120°$"; or "2 obtuse $\angle$s $= 240°$;" all of which mean precisely the same thing, since opposite $\angle$s of $\square$s are equal, and all 4 angles $= 360°$. (*Vid.* references in note above.)

Case IV. Given, vertical $\angle = 60°$, and *longer* diagonal: find area.

[*N. B.* A C being given, A E is known;

$$\text{and B D} = \text{A B}, \quad \therefore \text{B E} = \frac{\text{A B}}{2}$$

$$\therefore \text{A B}^2 - \text{B E}^2 = \text{A B}^2 - \frac{\text{A B}^2}{4} = \frac{3\,\text{A B}^2}{4} = \text{A E}^2.$$

$$\therefore \text{A B} = \sqrt{\frac{4\,\text{A E}^2}{3}}\ ;$$ and B D is therefore found immediately.]

The following are the *longer* diagonals of diamonds whose vertical $\angle = \frac{2}{3}$rds of a rt. $\angle$. Find respective areas.

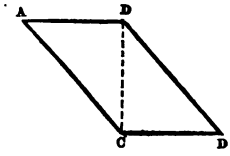
24 in.; 72 ft.; 108 yds.; 78 p.; 56 in.; 21 yds. 1 ft.

Ans. 1 ft. $22\frac{1}{2}$ in.; $166\frac{1}{2}$ yds.; 3367 yds.; $1756\frac{1}{2}$ p.; $905\frac{1}{2}$ p.; 1182·4 ft.

Additional Case of the Rhomboid.

When an oblong has been wrested out of the perpendicular just so far as to make 2 opposite ∠s together = 90°, the other 2 = 270°; so that while ∠ B A C = 45°, ∠ A B D = 90° + 45°, the *shorter* diagonal B C dividing it thus exactly. The ∠s A C B and C B D therefore each = 45°, and ∴ A B and C D each = B C.

Fig. 21.



(Euclid, I. 34. 29.; Rt. ∠d Δ, Notes 6. and 7.)

1. The acute ∠s of a rhomboid each = 45°, and the shorter diagonal is 19 inches. Find area. *Ans.* 2 ft. 73 in.

2. The acute ∠s of a rhomboid together = 90°, and shorter diagonal is 57 yds. Find area. *Ans.* 2 r. 27 p. 12¼ yds.

3. The obtuse ∠ of rhomboid = 135°, and shorter diagonal = 15 yds. 2 ft. 6 in. Find area. *Ans.* 8 p. 8 yds. 6¼ ft.

4. The obtuse ∠s of rhomboid together = 270°, and diagonal forming them is 66 p. 2 yds. 2 ft. 3 in. Find area. *Ans.* 26 a. 2 r. 22½ p.

5. One acute ∠ of rhomboid = ⅛ of 4 right ∠s, and shorter diagonal is 33 c. 25 lks. Find area. *Ans.* 100 a. 5 c. 5625 lks.

6. The 2 obtuse ∠s of rhomboid = ⅔ of 2½ rt. ∠s, and diag. joining them is 5 ft. 11¼ in. Find area. *Ans.* 3 yds. 8 ft. 108$\frac{1}{16}$ in.

Miscellaneous Questions on the Triangle, Trapezium, Rhombus, &c.

[*N. B.* Let diagrams be made as before.]

1. *A*, *B*, *C*, are fields shaped as trapeziums.
Diagonal of *A* is 57 yds. and ⊥s upon it are 29 and 37 yds.
Diagonal of *B* is 239 yds. and ⊥s upon it are 39 and 98 yds.
Diagonal of *C* is 468 yds. and ⊥s upon it are 97 and 285 yds.
Find area of each. *Ans.* 1881 yds.; 16371½ yds.; 89388 yds.

2. The parallel sides of trapezoid *A* are 27 and 38 yds., and ⊥ distance 23 yds.; of trapezoid *B*, 49 and 56 yds., and ⊥ distance 47 yds. Find area of each. *Ans.* 747½ yds.; 2467½ yds.

3. Trapezium *A* has diagonal 15 c. 15 lks., and ⊥s 7 c. 69 lks. and 9 c. 87 lks.

Trapezium *B* has diagonal 7 c. 07 lks., and ⊥s 13 c. 79 lks. and 17 c. 34 lks.

Find areas. *Ans.* 13 a. 3c. 0170 lks.; 11 a. 0 c. 0445½ lks.

4. Trapezoid *A* has //s 27 c. 34 lks. and 17 c. 58 lks., ⊥ distance 15 c. 64 lks.

Trapezoid *B* has //s 23 c. 43 lks. and 19 c. 79 lks., ⊥ distance 17 c. 35 lks.

Find areas. *Ans.* 35 a. 1 c. 2744 lks.; 37 a. 4 c. 9335 lks.

5. The diagonal of a sq. room is 37 ft. Find cost of carpet at 7*s.* 6*d.* per yd. 1¼ yd. wide. *Ans.* £22 16*s.* 4*d.*

6. A square field measures 347 yds. diagonally. Find its value at 10*s.* 1*d.* per sq. rod, and cost of fencing at 8*s.* 3*d.* a rod.

Ans. £1003 8*s.* 2*d.*; £73 12*s.* 2½*d.*

7. The diagonals of a diamond-shaped pane of glass are 11 and 9 inches. Find its area, and length of each side.

Ans. 49½ sq. inch, and 7·1063 in.

8. The diagonals of diamond *A* are 16 and 12 inches; of diamond *B* 15 and 13 in. Find area and sides of each.

Ans. { *A*, ⅜ ft.; 10 inch. *B*, 97½ in.; 9·924 inch.

9. The diagonals of a hatchment are 11 and 8 ft. Find expence of emblazoning it at 51*s.* 9*d.* a sq. yd. And also length of side.

Ans. £12 13*s.*; 6¼ ft.

10. A tower, 261 ft. above the water level, stands on the edge of a river. A line from its top to the opposite bank measures 194 yds. Find breadth of river. *Ans.* 520·19515 ft.

11. A wall is 31 ft. high. Ditch at its foot is 8 yards wide. What length of ladder will reach top of wall from outside of ditch?

Ans. 39·20459 ft.

12. Two steamers start at the same time from same place, *A* due north, *B* east. At the end of 16 days they are 1920 leagues apart. *A* sails 216 miles per day. Find *B.'s* speed per hour. (*N.B.* Euclid I, 47.) *Ans.* 12 miles.

13. Two vessels start at same time from same place, *A* south, *B* west. *A* sails 2016 miles in a week, *B* 1512 miles per week. Find their distance apart in 19 hours. *Ans.* 285 miles.

14. *A*, *B*, *C*, are towns standing in a △, right ∠d at *B*, which is distant 156 miles from *A* and 117 from *C*. Find cost of a straight line of rail from *A* to *C* at £5 9*s.* 4½*d.* per yd. *Ans.* £1876875.

15. *A*, *B*, *C*, are towns standing in a △, right ∠d at *B*, 249 miles from *A*, and 332 from *C*. Find cost of a straight line of rail from *A* to *C*, at £2 19*s.* 4½*d.* per yd. *Ans.* £2168375.

16. Trapezium *A* has diagonal 469 yds., ⊥s 235 and 187 yds.
Trapezium *B* has diagonal 457 yds., ⊥s 178 and 356 yds.

Find value of *A* at £80 13*s.* 4*d.* an acre, and of *B* at £60 10*s.* per acre. *Ans.* *A* £1649 6*s.* 4*d.*; *B* £1525 4*s.* 9*d.*

17. Area trapezoid *A* is 1495 yds., and //s 27 and 38 yds.: find its ⊥ height. *Ans.* 46 yds.

18. Area trapezoid *B* is 2467½ yds. ⊥ height 47 yds., and one of the // sides 49 yds. Find 2nd //. *Ans.* 56 yds.

19. Diagonal of a square room is 45 ft. Find cost of carpet at 5*s.* 7½*d.* a yd. 1¼ yd. wide. *Ans.* £25 6*s.* 3*d.*

20. Diagonal of a square field is 987 yds. Find value at 7*s.* 6¾*d.* a square rod, and cost of fencing at 3¾*d.* per ft. *Ans.* £6088 11*s.* 1½*d.*

21. A house is 65 ft. high, and street 38 ft. wide. Find a ladder to reach top of house from opposite side of street. *Ans.* 75·292 inch.

22. From the top of a tower on a river-bank a line to the opposite side is 358 ft. The river 334·716 ft. wide. Find height of towe

Ans. 127 ft.

23. The diagonals of a diamond-shaped garden are 79 and 84 yds. Find its value at £121 per acre, and cost of walling it round at 2*s.* 6*d.* a yard. *Ans.* £82 19*s.*; £28 16*s.* 6¾*d.*

24. Diagonals of diamond *A* are 9¾ and 11¼ yds., of *B* 79¾ and 63¼ yds.; of *C* 98½ and 75¾. Find area and side of each.

Ans. { *A*, 54$\frac{37}{64}$ yds.; 7·4 yds.
B, 2522$\frac{3}{32}$ yds.; 50·893 yds.
C, 3730$\frac{11}{16}$ yds.; 62·128 yds. }

25. Two ships start south and east from same place at same time. *A* steams 4032 miles in a fortnight, and *B* only 3024 miles. How far asunder are they at the end of 38 hours? *Ans.* 570 miles.

26. Two ships, making 6 knots an hour each, sail from the same port. *A* starts at noon due west. At 3 P.M. *B* sails north. Find their distance apart at midnight. *Ans.* 90 miles.

27. *A*, *B*, *C*, are towns situated at the ∠s of a △, right ∠d at *B*, which is sixty-eight miles from *A*, and 51 from *C*. Railway from *A* to *C* costs £3 18*s.* 9*d.* a yd., and is constructed with capital borrowed at 3¾ per cent. How much money will have been paid as interest in 8 years? *Ans.* £176715.

28. Diagonal of trapezium is 108 ft. 6 in., and ⊥s 56 ft. 3 in. and 60 ft. 9 in. Find its value at 3*s.* a yd. *Ans.* £105 15*s.* 9*d.*

21. Trapezoid *A* has //s 19 and 23 yds., and ⊥ 38 ft.
Trapezoid *B* has //s 59¼ and 47½ yds., ⊥ 38 yds.
Find value of each at £20 3*s.* 4*d.* per rood.

Ans. £4 8*s.* 8*d.*; £33 16*s.* 1*d.*

30. Diagonal of square field is 978 yds. Find value at £90 15*s.* an acre, and fencing at 7½*d.* a ft. *Ans.* £8967 0*s.* 9*d.*; £259 6*s.* 8½*d.*

31. Diamond tiles measure 10 and 7 inches diagonally, and [redacted]

5*s.* 3*d.* a dozen. How many will cover a space of 30 yds. 5 ft. 125 in., and what are they worth? *Ans.* 1135; £24 16*s.* 6¾*d.*

32. A fir tree being partially broken by the wind, at height of 38 ft. from the root, its top touches the ground at an ∠ of 45°. Find its whole height. *Ans.* 91·74 ft.

33. A fir tree being partially broken by the wind, its top touches the ground at an ∠ of 45°, and distance of 47 ft. from foot. Find its whole height. *Ans.* 113·468 ft.

34. A mast 98 ft. high casts a shadow of 76 ft. Find length of rope reaching from extremity of shadow to a point in the mast $\frac{5}{7}$ of its height from deck. *Ans.* 103·324 ft.

35. A square court whose diagonal is 48·2907 yds., is paved with 4,624 sq. tiles. Find dimensions of a tile. *Ans.* 18 inch sq.

36. Diamond quarries of glass measure 6 and 5 inches diagonally. How many will fill a window ($18 \times 12\frac{1}{2}$) ft., and what will they cost at 13*s.* 6*d.* a gross? *Ans.* 2160; £10 2*s.* 6*d.*

37. Diagonals of a hatchment are $9\frac{3}{4}$ and $7\frac{1}{2}$ ft. It is emblazoned at $3\frac{3}{4}$ guineas a square yd., and banded with iron along each side at 5*d.* a ft. Find whole cost. *Ans.* £16 10*s.* $2\frac{1}{16}$*d.*

38. *A* and *B* each get over 1008 miles in a week. *A* sails north at 8 A.M., and when she has gone 18 miles, *B* sails east from the same place. How far asunder will they be at 8 P.M.? *Ans.* 30 leagues.

39. *A* and *B* leave same port at same time. *A* sails west 1512 miles in a week, and *B* north 288 miles in a day. Find distance apart in 4 dys. 3 hrs. *Ans.* 1485 miles.

40. *A*, *B*, *C*, are towns standing in a △, right ∠d at *B*, which is 98 miles from *A*, while *A* is 137 miles from *C*. Find cost of railway from *B* to *C* at £4 17*s.* 6*d.* a yd. *Ans.* £821397 14*s.* 4½*d.*

41. Sides of a △r lawn are 95, 57, 76 yds. Find cost of laying it down with turfs (24 × 18) inch, and worth 2*s.* 6*d.* a score.
Ans. £40 12*s.* 3*d.*

42. A △r garden, right ∠d at *B*, which is 27 yds. from *A* and 36 from *C*, is divided by a wall running ⊥ upon *A C* from *B*. Find length of wall, and area of each portion of garden.
Ans. { Wall, 21·6 yds.
{ (1.) 174·96 yds. (2.) 311·04 yds.

43. Area trapezium *A* is 89388 yds, and ⊥s 285 and 97 yds.
Area trapezium *B* is 1881 yds., diagonal 57 yds., and one ⊥ 37 yds. Find diagonal of *A*, and 2nd ⊥ of *B*. *Ans.* 468 yds.; 29 yds.

44. Value of a trapezium at £60 10*s.* an acre is £1525 4*s.* 9*d.* Its ⊥s are 356 and 178 yds. Find diagonal. *Ans.* 457 yds.

45. Value of trapezium at £80 13*s.* 4*d.* an acre is £1649 6*s.* 4*d.* Diagonal is 469 yds., and one ⊥ 187 yds. Find the other. *Ans.* 235 yds.

46. A fence runs diagonally through a square field, and measures 576 yds. Find value of field, at £45 7*s.* 6*d.* per acre, and cost of fencing it at 1*s.* 3*d.* a yd. *Ans.* £1086 9*s.*; £85 2*s.* 1½*d.*

47. The side of a square meadow is 7906 ft. Find expense of a diagonal drain at 4*s.* 1½*d.* per rod. *Ans.* £559 nearly.

48. A maypole being snapped off in a storm, a triangle is formed by its top touching the ground at a distance of 28 ft. from its foot. The portion still upright is 23 ft. Find whole length. *Ans.* 59·235 ft.

49. A maypole being snapped off in a storm, a triangle is formed by its top touching the ground at a distance of 38 ft. from its foot, and the fallen piece measures 46·615 ft. Find whole length. *Ans.* 73·615 ft.

50. The hypothenuse of a right ∠d meadow is 295 yds., and one of the other sides is 177 yds. Find value at £75 12*s.* 6*d.* an acre, and cost of fencing at 8*s.* 3*d.* a rod. *Ans.* £326 6*s.* 10½*d.*; £53 2*s.*

51. Hatchment *A* measures 9⅓ and 7¾ ft. diagonally, and *B* 9¾ and 8½ ft. Find cost of emblazoning each at 2½ guineas a square yard. *Ans.* £10 14*s.* 8¾*d.*; £12 1*s.* 8⅝*d.*

52. Find length of a side in each of the foregoing diamonds. *Ans.* 6·13 ft.; 6 467 ft.

53. Two steamers start at same time from same place north and west, and in 5¼ days are 2520 miles apart. One travels 12 miles an hour. Find speed of the other. *Ans.* 16 miles.

54. *A*, *B*, *C*, stand at the ∠s of a △ right ∠d at *B* which is 69 miles from *A* and 92 from *C*. Railway from *A* to *C* costs £796950. How much a yd. is this? *Ans.* £3 18*s.* 9*d.*

55. *A*, *B*, *C*, stand at the ∠s of a right ∠d △ whose sides are in the proportion of 3, 4, 5. Cost of rail from *A* to *C* at £6789 17*s.* 6*d.* a mile is £577139 7*s.* 6*d.* Find distances apart. *Ans.* 51, 68, 85 miles.

56. *A*, *B*, *C*, stand at the ∠s of a right ∠d △ whose sides are in the proportion of 3, 4, 5. Cost of rail from *A* to *B* at £5670 a mile is £1003590. Find distances apart. *Ans.* 295, 236, 177 miles.

57. A room is a regular trigon, one of whose sides is 5⅓ yds. Floor it with carpet at 4*s.* 5¾*d.* per yd., 43 inches wide. *Ans.* £2 6*s.* 2¼*d.*

58. Find value of a triangular piece of ground, two of whose sides are 40 yds. and the other 48 yds., at 6¾*d.* a square ft. *Ans.* £194 8*s.* 0*d.*

59. Diagonals of a hatchment are 9 ft. 9 in. and 7 ft. 9 in. Find cost of emblazoning it at 3¾ guineas a square yd., and banding each side with iron at 1*s.* 6*d.* a yd. *Ans.* £17 3*s.* 0½*d.*

60. The sides of a triangular garden are 58, 64, 76 yds. Find value at 7*s.* 6¾*d.* a sq. pole. *Ans.* £22 11*s.* 11*d.*

61. A □ic field has two opposite angles = 240°, and diagonal joining them = 180 yds. Find value at £15 2*s.* 6*d.* a rood. *Ans.* £350 15*s.* 3*d.*

62. A ⊐ic field has one angle = 120°. It is divided by a hedge running diagonally from this angle, and measuring 456 yds. Find value at £80 13*s.* 4*d.* an acre, and fencing at 6*s.* 2½*d.* a rod.

Ans. £3001 5*s.* 10½*d.*; £102 12*s.*

63. A ⊏ic field has two opposite angles = 270°, and diagonal between them = 678 ft., which is also the length of the shorter side. Find value at 7*s.* 6½*d.* a sq. rod, and fencing at 4*s.* 1½*d.* a rod.

Ans. £638 9*s.*; £40 18*s.* 5*d.*

64. Diagonal of a trapezium is 79 c. 85 lks. and perpendiculars 29 c. 76 lks., 38 c. 54 lks. Perpendicular of a trapezoid is 57 c. 57 lks. and parallels 96 c. 73 lks., 68 c. 37 lks. Find area of each.

Ans. { 272 a. 6 c. 8775 lks.
{ 475 a. 2 c. 4035 lks.

65. The parallel sides of a trapezoid are 179 and 98 yds., and perpendicular height 258 yds. Find joint length of the 2 sides which are not parallel, (1.) when equally inclined to each other, (2.) when one is perpendicular to the parallels. *Ans.* 528·4 yds.; 522·3 yds.

66. The diagonals of 2 sq. rooms are 23¾ and 49¾ ft. Carpet, 1¼ yd. wide, is laid down in each. For the former it is 11*s.* 3*d.* a yd. and the latter 5*s.* 7½*d.* do. Find cost in each case.

Ans. £14 2*s.* 0⅜*d.*; £30 18*s.* 9⅛*d.*

67. At £80 13*s.* 4*d.* an acre, a square meadow is worth £2713 13*s.* 6*d.* Find length of a diagonal drain. *Ans.* 579 yds.

68. A roof of 16 ft. span forms a right angle at the ridge. Find cost of weather-boarding for the two sloping edges at 7*s.* 6*d.* a yd.

Ans. £2 12*s.* 6½*d.*

69. A ladder 38 ft. long leans, at an angle of 45°, against a wall 28 yds. wide. Find height of wall, and cost of painting it at 6*s.* 9*d.* a sq. yd. *Ans.* 26·869 ft.; £84 12*s.* 9*d.*

70. Diamond quarries are 11 and 8 inches diagonally, and cost 2*s.* 9*d.* a dozen. Find expense of glazing a window ($23\frac{1}{2} \times 13\frac{1}{2}$) ft.

Ans. £71 7*s.* 3⅜*d.*

71. The emblazoning of a hatchment costs £10 14*s.* 8¾*d.* Its diagonals are 9 ft. 6 in. and 7 ft. 9 in. How much is this per sq. yd. ?

Ans. £2 12*s.* 6*d.*

72. A barn, 24 yds. × 28 ft., has two gable ends, each 19 ft. from floor to eave, and 18 ft. from eave to ridge, perpendicular height. Find expense (1) of plastering the ends at 1*s.* 10½*d.* a sq. yd., (2) of slating the roof at 11½*d* a sq. yd. *Ans.* P, £16 6*s.* 8*d.*; S, £67 5*s.* 6½*d.*

73. A square meadow contains 119025 sq. yds. Find cost of fencing it at 8*s.* 3*d.* a rod, and of making two diagonal drains at 3¾*d.* a ft.

Ans. £103 10*s.*; £45 14*s.* 9¾*d.*

74. The painting of a hatchment at 1½ gs. a sq. yd. costs £5 16*s.* 4½*d.* One of its diagonals is 7 ft. Find the other. *Ans.* 9 ft. 6 in.

75. $A\,B\,C$ is a triangular field right-angled at B, which is 141 yds. from A, and 188 yds. from C. It is divided into two portions by a straight hedge which runs from B at right angles to AC. Find value of each portion at 7*s.* $6\frac{3}{4}d.$ per sq. rod.

Ans. £59 12*s.* $10\frac{1}{4}d.$; £106 0*s.* $7\frac{3}{4}d.$

76. A square slate, area 6 ft. 36 in., has a frame 2 in. wide. Find frame's outside dimensions and cost at 3*d.* a ft.

Ans. 2 ft. 10 in.; 2*s.* 8*d.*

77. A and B start east and south from the same place at 6° 30′ A.M., at rates of $3\frac{3}{4}$ and $4\frac{1}{4}$ miles per hour respectively. How far asunder will they be at 9° 15′ P.M., allowing to each a pause of two hours at midday for rest and refreshment? *Ans.* 72·265 miles.

78. Three houses stand in a triangle, right-angled at B, which is 176 yds. from C, and 189 from A. In walking from A to C, how much is saved by going direct, instead of round by B? *Ans.* $106\frac{3}{4}$ yds.

79. A, B, C, stand in a triangle, right-angled at B, whose sides are as 3, 4, 5. Money is borrowed at £3 15*s.* per cent. to construct a railway from B to C, which costs £3 8*s.* 9*d.* a yd. Annual interest on the capital is £32670. Find distances apart. *Ans.* 144, 192, 240 miles.

80. Find expense, at $13\frac{1}{2}d.$ a sq. yd., of gravelling a triangular yard whose base is 16 yds., and sides 40 ft. each. *Ans.* £6 8*s.*

81. The diagonal of a ▱ic garden is 98 yds., and joins two angles which equal 240°. Find value of garden at 12*s.* $7\frac{1}{4}d.$ a rod.

Ans. £173 5*s.* $6\frac{1}{4}d.$

82. A ▱ic field has two opposite angles together equal to 120°, while the other two are joined by a straight line which is 798 ft. Find its value at 12*s.* $7\frac{1}{4}d.$ a sq. rod, and fencing at 12*s.* $4\frac{1}{2}d.$ a rod.

Ans. £1253 19*s.* $8\frac{3}{4}d.$; £119 14*s.*

83. In attempting to estimate the breadth of an unfordable river, I stand at point A, directly opposite a tree, B, on the other side. I then walk in a direction at right angles to the imaginary line AB, until at C, a distance of 86 yds. from A, I observe $\angle ACB = 45°$. Show that 86 yds. is the river's breadth. (*Vid.* Appendix.)

84. A and B are trees on opposite sides of an impassable quagmire. Wishing to measure the distance between them, I find a point C exactly 175 yds. from both A and B, and observe $\angle\ ACB = 60°$. Show that $AB = 175$ yds.

85. A field is a trapezium with diagonal 279 yds. and perpendiculars 178, 189 yds. Each pair of adjacent sides is of the same length. Find them. *Ans.* 234·906 and 225·15 yds.

86. A field is a trapezoid, having its perpendicular height 197 yds. and the 2 parallel sides 127 and 315 yds. Find the joint length of the other 2 sides, (1.) when equally inclined to each other, (2.) when one of them is perpendicular to the parallels. *Ans.* 469·31 and 436·554 yds.

87. The fencing of a square field at 4*s.* 1½*d.* per rod costs £51 15*s.* Find cost of a diagonal drain at 1*s.* 4¾*d.* a ft. *Ans.* £102 3*s.* 1½*d.*

88. An expense of 17*s.* 0¾*d.* is incurred for veneering a square table at 3*s.* 9½*d.* a sq. ft. Find length of its diagonal. *Ans.* 1 yd.

89. A barn is 49 ft. long. Its roof is 32 ft. span, rising four yds. perpendicular from the eave, and making a right angle at the ridge. Find cost of slating it at 11¼*d.* a sq. yd. *Ans.* £10 4*s.* 2*d.*

90. The hypothenuse of a right-angled meadow is 295 yds. Find its value at £75 12*s.* 6*d.* an acre. *Ans.* £326 6*s.* 10½*d.*

91. A ladder 49 ft. long being planted 3½ yds. from a house reaches to ¾ of its height. How much longer must it be to reach the top?

Ans. 15·472 ft.

92. A □ic meadow has 2 drains which run straight across between opposite corners, measuring 180 and 310·9 yds. Find its value at £60 10*s.* per acre, and fencing at 9*s.* 10¼*d.* a rod.

Ans. 349 15*s.* 3*d.*; £64 10*s.*

93. *A*, *B*, *C*, stand in a triangle right-angled at *B*, whose sides are as 5, 4, 3. Rail from *A* to *B* costs £5670 a mile, and yearly interest at 5 per cent. on capital borrowed to construct the line, is £50179. Find distances apart. *Ans.* 295 &c.

94. How much gold leaf will be needed for the edging of a book (18 × 13) inches and 2¾ inches thick? *Ans.* 121 inches.

95. The hypothenuse of a right-angled field is 335 yds. Find value at 7*s.* 6¾*d.* a sq. rod. *Ans.* £336 13*s.* 6*d.*

96. The side of a diamond piece of ground, whose vertical angle = 60°, is 79 yds. How many persons may have gardens 10 yds. square out of it? *Ans.* 54.

97. Area of an isosceles triangle is 84 yds., and perpendicular height 14 yds. Find base and sides. *Ans.* 12 and 15·23 yds.

98. The fencing of a sq. field at 7½*d.* a ft. costs £259 6*s.* 8½*d.*, and 2 ditches cross it diagonally. Find expense of digging them at 7½*d.* a ft.

Ans. £183 7*s.* 6*d.*

99. Area △ is 361 yds., and perpendicular height divides base into 2 portions which are as 8 : 11. Find 3 sides.

Ans. 38, 29·068, 24·839 yds.

100. The base of a right-angled triangle field is 300 yds., and sum of the other sides is 1000 yds. Find cost of fencing each at 3*s.* 10¾*d.* a rod.

Ans. { £10 12*s.* 6*d.*; £16 2*s.* 3½*d.*; £19 6*s.* 0½*d.*

101. Paving of a triangular court, at 4*s.* a yd., costs £26 13*s.* 4*d.* The longest side is 80 ft. Find sum of other 2. *Ans.* 100 ft.

102. The shorter of 2 parallel sides of a trapezoid is 78 yds., and perpendicular ht. 65 yds. The other 2 sides are 87 and 96 yds. Find second parallel. *Ans.* 206·474 yds.

103. The longer of 2 parallel sides of a trapezoid is 97 yds., perpendicular 48 yds., and the others 68 and 54 yds. Find area.

Ans. 2896·304 yds.

104. The diagonals of squares *A*, *B*, *C*, are 6, 12, 18 inches. Show that their areas are as 1, 4, 9.

105. Mast of ship is 97 ft. high, and deck 86 ft. wide. Find cost of rope-ladder from ship's side to cross-trees, which are 68 ft. above deck, at 2*s.* 6*d* a yd. *Ans.* £3 7*s.* $0\frac{39}{50}d.$

106. A triangular garden has a straight walk which bisects the vertical angle, and divides wall at base into two portions of 72 and 58 yds. Joint length of the other 2 sides is 270 yds. Find area. (Eucl. VI. Prop. B.) *Ans.* 7496·055 yds.

107. From a right-angled triangle, base 18 ft., perpendicular 24 ft., a triangle (area 6 sq. yds.) is cut off by a line parallel to perpendicular. Find its sides. (Eucl. VI. 2.) *Ans.* 3, 4, 5 yds.

108. Find side of equilateral triangle whose area = that of a square with side $3\frac{1}{2}$ yds. *Ans.* 15·196 ft.

109. A right-angled triangle garden contains 96 sq. rods. Its longest side is 5 chains. Find the other two. *Ans.* 3 and 4 chains.

110. Value of a garden, (isosceles triangle) at 7*s.* $6\frac{3}{4}d.$ a sq. pole is £34 1*s.* 6*d.*, and its perpendicular length 47 yds. Find sides.

Ans. 116 and 74·652 yds.

111. A right-angled piece of ground has its perpendicular sides 21 and 28 yds. A fence running parallel to latter cuts off a portion containing 96 sq. yds. Find its sides. (Eucl. VI. 2.) *Ans.* 16, 20, 12 yds.

112. A triangular meadow *A*, *B*, *C*, right-angled at *B*, has its sides *A B* and *B C* = 188 and 141 yds. A drain is dug from *B* perpendicular to *A C* at cost of 5*s.* 4*d.* a yd. Find its cost, and value of the portions of ground on each side of do. at £60 10*s.* per acre.

Ans. { Drain, £30 1*s.* $7\frac{1}{2}d.$ / Meadow, £106 0*s.* $7\frac{1}{2}d.$; £59 12*s.* $10\frac{1}{2}d.$

113. *M* and *N* start at same time from angle *A* of equilat. triangle *A B C* to go round it, *M* along *A B*, *N* along *A C*. *M* travels 3 times as fast as *N*. Where will they meet? *Ans.* $\frac{3}{4}$ *AC*.

114. A snail travels up a tree 70 ft. high, 7 ft. every day, but crawls down a yd. every night. How soon will he reach the top?

Ans. $16\frac{3}{4}$ days.

115. A meadow (120 × 84) yds. is to be drained with tiles 18 inches long, laid in rows 9 ft. apart. Find cost, at 15*s.* per thousand.

Ans. 41*s.* 8*d.*

116. A triangular sail, sides 13, 14, 15 yds., loses a triangular piece containing 24 sq. yds., which is ripped off in a line running parallel to sail's longest side. Find sides of this triangle.

Ans. 6·948, 7·483, 8·017 yds.

117. *A B C* is a triangular garden, sides 43, 57, 80 yds. A straight wall crosses it at right angles to the longest side from the angle opposite, and costs 6*s*. 3*d*. a yd. This expense is borne by *M* and *N*, who respectively own the portions on either side of the wall, and who pay in proportion to their size. Find value of each portion, at £90 15*s*. per acre, and expense of wall to each.

Ans. { *M*, £8 13*s*. Wall, £3 11*s*. 7*d*. nearly.
{ *N*, £13 9*s*. 11¾*d*. Wall, £5 12*s*. 11*d*. do.

118. Two triangles have each a perpendicular height of 40 yds.; but the base of one of them is five yds. less than that of the other, whose area is ¼ longer in consequence. Find bases. *Ans.* 20 and 25 yds.

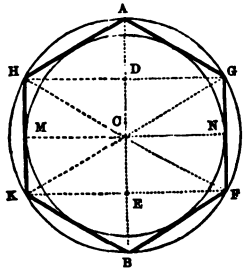
119. A person being asked the area of his field (a reg. 3^{n}) answered — "If to half the length of one side you add its sq. root, and subtract 60 yds. from the sum, there will remain no side at all." Determine the area from this datum. *Ans.* 4330 yds.

120. *A* and *B* together own 14 acres of land, *A*'s share being ⅔ths of *B*'s. *C*, having none, wishes to purchase of *A* and *B*, so that all three may have equal allotments. At £60 per acre, show that *A* will receive £80, and *B* £200.

THE REGULAR POLYGON.

1. Any figure having 3 or more angles may be called a polygon (πολυ, several, and γονυ, knee, corner); the number of *sides* of course being the same as the number of ∠s. And its name depends upon the number of ∠s it contains,— being termed a trigon, tetragon, pentagon, hexagon, heptagon, &c., according as its ∠s are 3, 4, 5, 6, 7, &c.

Fig. 22.



2. When all the angles, and therefore all the sides, are equal, the polygon is called "regular;" and *then* it is such, that from its centre circles may be both *inscribed* in it and *circumscribed* about it,—the former touching each *side* exactly in the middle, and the latter passing through each *angle*.

(*Vid. Fig.* above, and Euclid, III. 18.)

3. Every regular polygon may be divided into as many equal triangles as it has angles or sides, by drawing straight lines from each $\angle$ to the centre of the figure (Euc. I. 32.); *i. e.* a polygon of 6 sides will contain 6 Δs, and one of x sides, x Δs. (*Vid. Fig.*)

4. The area, therefore, of any reg. polygon may be easily calculated whenever a *side* is known, and also the perpendicular upon it, from the centre (*i. e.* the radius of the inscribed circle). For if it has x sides, each a ft. long, and the $\perp$ from the centre is β ft. long, then the area of each $\Delta = \frac{a \times \beta}{2}$; and $\therefore$ the area of all the x Δs, *i. e* of the whole polygon, $= x$ times $\frac{a \times \beta}{2} = \frac{x a \beta}{2}$.

5. x times a, or $x a$, *i. e.* the *sum of the sides* of the above polygon, or their total length, is called the "perimeter."

6. It is not, however, always possible to see and count the number of *sides* in a polygon, whose area has to be determined notwithstanding. And it will be well, therefore, to consider how this may be accomplished, when only the size of one or more *angles* can be ascertained. It is done as follows.

7. Since the 3 $\angle$s of every $\Delta = 180°$ (Rt. $\angle$ d Δ, Note 2.), it follows that the $\angle$s of all the x Δs in the polygon together $= x$ times 180°, or $180° x$. Now the angles at the *centre* of polygon, which is the common vertex of all the Δs, together $= 4$ rt. $\angle$s or 360°, (Eucl. I. 15.), $\therefore$ the $\angle$s at the *base* of the Δs, *i. e.* the x $\angle$s of the pol. $= 180° x - 360°$, or $180° (x-2)$; and $\therefore$ each $\angle$ of pol. $= \frac{180° (x-2)}{x}$.

Ex. 1. The only angle visible of a certain unknown regular polygon is one of 135°. Find the number of sides.

Let x = number of sides or angles.

Then $135° x$ = number of degrees in all the x angles of polygon $= 180° x - 360°$.

$\therefore x = 8$, and the *fig.* is an Octagon.

Ex. 2. The only angle visible of do. do. is an *exterior* one of 72° Find number of sides.

Now the adjacent *interior* angle of polygon = 180° − 72° = 108° (Euclid, I. 13.)

∴ as before, $108° x = 180° x - 360°$, and the *fig.* is a Pentagon.

Ex. 3. Find value of each angle in a reg. Hexagon.

Here $x = 6$; ∴ each angle $= \frac{180° (6-2)}{6} = 120°$.

Ex. 4. The sum of the angles of a polygon = 720°. Find the number of sides.

Here x angles $= 720° = 180° x - 360°$.

∴ $x = 6$, and the *fig.* is a Hexagon.

8. The area of a reg. polygon varies, *i. e.* increases or diminishes, as the *square* of its side or diameter. Thus: if the *side* or *diameter* be doubled, trebled, &c., the *area* becomes 4 times, 9 times, &c., what it was before, and *vice versâ*. For if *side*, *i. e. perimeter*, be doubled, &c., then *diameter* is of necessity doubled, &c. too, and *vice versâ*. And *area* depends upon the *product of both*, or the square of either.

9. Take, for illustration, the simplest reg. figure, viz. the square, and "*ex unâ disce omnes*," since what is true of one is (*mutatis mutandis*) true of all alike. Let the square's side = 3 ft., then diameter or ⊥ ht. = 3 ft. also, and ∴ area = $3 \times 3 = 9$ sq. ft. Now if *side* be doubled, *i. e.* $= 3 \times 2$, then diam. $= 3 \times 2$, and ∴ area $= (3 \times 2) \cdot (3 \times 2)$, *i. e.* (9×2^2) or 6^2, which is not *twice* but *four times* its former size. And so of any other increase or diminution in height or perimeter.

Case I. Given, side of polygon and ⊥ from centre, *i. e.* the radius of the inscribed circle. Find its area.

[*N.B.* Area polygon $= \frac{\text{perimeter}}{2} \times$ perpendicular from centre. *Vid.* Notes 4 and 5., and Euclid, III. 18.]

	No. of Sides.	*Length of Side.*	⊥ *from cr.*	*Ans. Area.*
1.	3.	53 in.	15 in.	8 ft. 40½ in.
2.	4.	56 ft.	28 ft.	348 yds. 4 ft.
3.	5.	3 ft.	25 in.	1 yd. 6 ft. 90 in.
4.	6.	30 p.	26½ p.	14 a. 3 r. 25 p.
5.	8.	23 c. 75 lks.	28 c. 25 lks.	268 a. 3 c. 7500 lks.
6.	10.	19 in.	29 in.	2 yds. 1 ft. 19 in.
7.	12.	31 yds.	59 yds.	2 a. 1 r. 2 p. 23½ yds.
8.	5.	2 c. 16 lks.	1 c. 50 lks.	8 c. 1000 lks.

ɪse II. Given, side and line joining the centre and ɔ, *i. e.* the radius of the circumscribed circle. Find area ɔlygon.

V.B. ⊥ C N is found from the equation $CF^2 = NF^2 +$ *vid. fig.*]

o. Sides.	*Length of Side.*	*Radius.*	*Ans. Area.*
3.	8 ft. 10 in.	5 ft.	3 yds. 4 ft. 7·716 in.
5.	63 yds.	54 yds.	1 a. 1 r. 28 p. $10\frac{3}{4}$ yds.
6.	7 c. 50 lks.	165 yds.	14 a. 6 c. 0250 lks.
8.	47 p.	60 p.	64 a. 3 r. $18\frac{1}{2}$ p.
5.	1 c. 05 lks.	90 lks.	1 c. 9189 lks.
3.	4 yds. 1 ft. 3 in.	2 yds. 1 ft. 6 in.	7 yds. 6 ft. 128 in.
8.	141 yds.	180 yds.	19 a. 1 r. 11 p. 6·336 yds.
6.	45 fathoms.	270 ft.	4 a. 1 r. 2 p. 13·93 yds.
10.	38 ft.	20 yds.	1201·476 yds.
12.	62 yds.	120 yds.	43114·8 yds.

ɪse III. Given, radii of inscribed and circumscribed ɔs : find area of polygon. (*Vid.* Note to Case II.)

No. Sides.	*Rad. inscr.*	*Rad. circ.*	*Ans. Area.*
4.	43 yds.	61 yds.	7396 yds.
8.	28 ft.	$30\frac{1}{2}$ ft.	2709 yds.
6.	$21\frac{3}{4}$ ft.	25 ft.	178·727 yds.
12.	97 ft.	100 ft.	3144 yds.
5.	$19\frac{1}{2}$ ft.	24 ft.	151 yds. $5\frac{1}{3}$ ft.
5.	25·24 ft.	31 ft.	252 yds. $3\frac{1}{3}$ ft.
9.	$22\frac{1}{3}$ yds.	24 yds.	1766 yds. $2\frac{1}{4}$ ft.
8.	28 yds.	31 yds.	2981 yds.
10.	$22\frac{3}{4}$ yds.	24 yds.	1739 yds.
12.	29 yds.	31 yds.	3812 yds.

ɪse IV. Given, the number of degrees in one or more ɪf a polygon : find number of sides, and thence the area. . Note 7. and examples.)

The interior angles of 5 polygons contain respectively 140° ; 60° ; 90° ; $128\frac{4}{7}$°. Find the polygons to which each belongs.

Ans. 9gon; 3; 10; 4; 7.

The exterior angles of 5 polygons contain respectively 60° ; 45° ; 24° ; $32\frac{8}{11}$°. Find polygons. *Ans.* 6; 8; 12; 15; 11.

3. Find value of each angle in polygons which have 5; 15; 20; 18; 24 sides respectively. *Ans.* 108°; 156°; 162°; 160°; 165°.

4. The sum of the angles in 5 polygons respectively is 540°; 900°; 1260°; 1800°; 1980°. Find the polygons. *Ans.* 5gon; 7; 9; 12; 13.

5. Find difference between the exterior angle of a pentagon and the interior angle of an undecagon. *Ans.* $95\frac{40}{77}$°.

6. Find number of degrees in the exterior angle of a 12gon, 8gon, and 17gon respectively. *Ans.* 30°; 45°; $21\frac{3}{17}$°.

7. The angles of a pentagon are as the numbers 4, 5, 6, 7, 8. Find them. *Ans.* 72° &c.

[*N. B.* $4+5+6+7+8=30$; $\therefore$ the least angle $= \frac{4}{30}$ of 540°, &c. &c.]

8. The angles of a hexagon are as the numbers 4, 6, 8, 10, 12, 14. Find them. *Ans.* $53\frac{1}{3}$° &c.

9. The angles of a pentagon are in arithmetical progression, (like those in questions 7 and 8) and the second smallest is $\frac{3}{4}$ of the angle of a hexagon. Find them. *Ans.* The largest is 144°; &c.

[*N. B.* Let x = common difference; then the least angle $= 90° - x$, the third $= 90° + x$, &c.]

10. The angles of a pentagon are in arithmetical progression, (like those in questions 7 and 8) and the second largest is $\frac{13}{18}$ of the angle of a 10gon. Find them. *Ans.* The least is 54° &c.

THE HEXAGON.

1. The hexagon, or 6-sided polygon (*vid. fig.* "Reg. Polygon") is a highly important figure, on account of the variety of ways in which it can be turned to account. The cells in a honeycomb are always of this shape, combining, as it does, convenience of form with economy of space and strength or power of resisting pressure. (*Vid.* Note 5, below, and Euel. IV. 15.)

2. The hexagon is very easily constructed thus. Take any straight line A B (*fig.* Reg. Polygon) and bisect it in C. With C as centre, and C A or C B as radius, describe circle. Again, bisect C A and C B in D and E, and through them draw H D G and K E F at rt. $\angle$s to A B, and meeting the $\bigcirc$ce of $\odot$ both ways in H, G, K, F. Join the points A, H, K, B, F, G, and the result is a reg. 6gon.

3. The *sides* of the 6gon being equal, the *angles* opposite to or subtended by them at centre C must be equal too,—*i. e.* ∠ A C G = ∠ G C F = ∠ F C B = &c. And since all the 6 ∠s at centre C together = 360°, (Reg. Polygon, Note 7.), each one of them must = $\frac{360}{6}$ = 60°; and ∴ each of the other ∠s in every Δ = 60° also; *i. e.* each of the 6 Δs is equiangular, and ∴ equilateral. [Hence each ∠ of 6gon = 120°.] (Rt. ∠d Δ, Notes 8. and 9.)

4. The hexagon therefore may be constructed thus:—take any straight line B F (*fig.* Reg. Polygon) and with a "protractor" (*vid.* Appendix) make the ∠s F B C and B F C each = ½ of 120°, or 60°. From the point C where the lines meet, with C B or C F as rad. describe ⊙, and then with the compasses lay down B F 6 times within the ○ce, as in figure. It will be observed that the opposite sides are parallel.

Or simply thus:—On any given straight line B F describe an equilateral Δ C B F (Eucl. I. 1.). With centre C and radius C B or C F describe a ⊙ and lay down B F 6 times within it.

5. The ∠ of a 6gon being 120°, this forms one of the 3 regular figures which alone can be used in covering a floor or otherwise filling up an area, the other 2 being the square or tetragon, and equilateral triangle or trigon. (*Vid.* Pott's Euclid, Bk. IV. notes.)

6. For since all the ∠s at any one point = 360°, (Eucl. I. 15.) no figures can possibly fit close together whose ∠s will not, at the point of junction, form a sum total of 360°; *e. g.*, pentagons will not answer the purpose, since each ∠ = 108°, and 3 times 108 is less, while 4 times 108 is more, than 360. And in the same way it may be shown that 7gons, 8gons, &c., can *not*, while hexagons, squares, or equilateral Δs *can*, fill up the space round a point; *i. e.* either 3 hexagons, 4 tetragons, or 6 trigons: since the sum of 360° is made up indifferently of either 3 times 120°, 4 times 90°, or 6 times 60°.

7. Since the hexagon consists of 6 equilateral Δs (Note

3.) its area may be determined not only as any other regular polygon, but also when a *side only* is known, (*vid.* Scalene Δ) or the *diagonal*, which is just twice the length of a side. By joining alternate angles of the 6n, a regular 3n is formed.

8. Observe the difference between "diagonal" and "diameter." The latter bisects a figure by joining either sides or $\angle$s; the former, by joining $\angle$s alone, whence its name (*γονυ*); *e. g.* (*fig.* Regular Polygon) A B and M N are both diameters, but only A B is diagonal. Hence curved figs., as the circle and ellipse, which have no $\angle$s, cannot have *diagonals*, but *diameters*.

Given, one side of a hexagon: find area. (Note 7.)

Ex. Each of the following lengths is one side of a 6gon.

60 yds.; 30 yds.; 19 yds.; 4·75 ft.; 76 p.; 1 c. 71 lks.; 48 in.; 21 ft., 36 yds.; 1 ft. 6 in.

Answers.

9352½ yds.; 2338½ yds.; 937·9 yds.; 6 yds. 4 ft. 89$\frac{1}{10}$ in.; 93 a. 3 r. 6$\frac{3}{4}$ p.; 7 c. 5970 lks.; 41·568 ft.; 127·302 yds.; 2 r. 31 p. 9$\frac{1}{4}$ yds.; 5 ft. 121$\frac{3}{4}$ in.

Case II. Given, area of 6n: find side.

The student may practise upon the areas found in Case I, according to the method suggested in Case II. of the equilateral Δ.

Case III. Given, side of 6n: find area of the 3n formed by joining its alternate $\angle$s. (Note 7.)

14 yds.; 9½ yds.; 128 in.; 13½ in.; 2 c. 56 lks.; 23½ p.

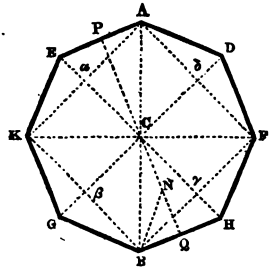
Answers.

254·598 yds.; 113·155 yds.; 16·42 yds.; 1 ft. 86·94 in.; 8 c. 5134 lks.; 4 a. 1 r. 27½ p.

THE OCTAGON.

1. The Octagon, or 8-sided figure, has its opposite sides parallel; and when its opposite angles are joined by diagonals, as A B, E H, &c., it is found to contain 8 isosceles Δs, as A C E, B C H, &c., all of the same dimensions.

Fig. 23.



2. When the *diameter* P Q is known, area is determined as in Regular Polygon, (Note 4.)

3. The Octagon may be constructed in any of the following ways.

a. Take any straight line A B (*vid. fig.*) and bisect it in C. With centre C and radius C A describe a ⊙. Through C draw K C F at rt. $\angle$s to A B, and join A K, K B, B F, F A. This is a square. Now bisect each side of this square in the points α, β, γ, δ, and join them, producing these *diameters* (*vid.* Hexagon, Note 8.) of the square both ways to meet the ⊙ in E, G, H, D. Join A E, E K, &c., and the result is a regular 8n.

β. Or thus: — Since the $\angle$ of an $8n = \frac{180\,(8-2)}{8} = 135°$ (Reg. Polygon, Note 7.), take any line A D, and at A and D make the $\angle$s A D C, D A C, each $= \frac{135}{2}$ or $67\frac{1}{2}°$. Then from centre C, where the lines meet, and with C A or C D as radius, describe a ⊙. Produce A C and D C to B and G, and then draw K F and E H $\perp$ to A B and D G respectively. Join A D, D F, &c., and a reg. 8n is formed.

γ. Or thus: — having described a ⊙ as before, with compasses lay down A D within it 8 times.

3. Or thus: — (nothing being needed but half of a square card to make an ∠ of 45°). Take any line G B (*fig. Case* III.) and draw G A, B D (of any sufficient length) ⊥ to it. Then lay down G F, B H, each = G B, and making an ∠ of 45° with G A, B D (Note 2.) Draw F E, H K // to G A, B D, and = G B. Similarly, draw E A, K D // to B H, G D. G A and B D will then be cut in A and D, which being joined the 8n is complete.

4. Or thus, if B H be the line given upon which to describe an 8n.

Bisect B H in Q and erect ⊥ Q N = B Q. Join B N and produce Q N to C, making N C = N B, and join B C, H C.

Then ∠ B N Q = 45° ∴ ∠ B N C = 135° ∴ ∠ B C N = 22½° ∴ ∠ B C H = 45°. (Rt. ∠d △ Notes 5—7).

With centre C and radius C B, describe ⊙, and a reg. 8n will be formed by laying down B H 8 times within the ⊙ce. (Reg. Pol. Note 7.)

5. If the alternate sides A D, F H, &c. be produced both ways, the result is a square, which will be found useful in *Case* III.

Case I. Given, side and half the bisecting diagonal, or radius of circumscribed ⊙, *i. e.* A D and A C: find area.

	Side.	*Radius.*	*Ans. Area.*
1.	60 yds.	79 yds.	17539·68 yds.
2.	40 yds.	52 yds. 2 ft.	7795 yds. 3 ft. 103½ in.
3.	2 c. 40 lks.	3 c. 16 lks.	2 a. 8 c. 0634 lks., and 55⅓ in.
4.	5 ft.	6 ft. 7 in.	13 yds. 4 ft. 115·68 in.
5.	26⅔ yds.	35⅓ yds.	3464 yds. 5·6532 ft.
6.	8 yds. 1 ft.	10 yds. 2 ft. 11 in.	338 yds. 3 ft. 12 in.

[If a hint be needed, *vid.* Reg. Pol. *Case* II.]

Case II. Given, side of 8n and line joining alternate angles, *i. e.* alternate diagonal or side of interior square: find area.

	Side of 8n.	*Side of Square.*	*Ans. Area.*
1.	24 yds.	45 yds.	2776·644 yds.
2.	4 ft.	7½ ft.	8 yds. 4 ft. 42 in.
3.	18 ft.	33 ft.	1563·7842 ft.

	Side of 8n.	*Side of Square.*	*Ans. Area.*
4.	1 c. 92 lks.	3 c. 60 lks.	1 a. 7 c. 7705 lks.
5.	72 p.	135 p.	156 a. 0 r. 29½ p.
6.	5 yds. 1 ft.	10 yds.	135 yds. 5 ft. 96 in.

[*N.B.* Area 8n = area (Square A F B K + 4 △ A D F).]

Case III. Given, side only: find area.

12 in.; 16 ft.; 24 yds.; 20 in.; 34 ft.; 42 yds.; 48 ft.; 64 yds.; 18 in.; 54 yds.

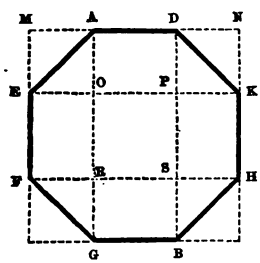
Answers.

4 ft. 119·2896 in.; 137 yds. 3·0704 ft.; 2781·1584 yds.; 1 yd. 4 ft. 59·36 in.; 620 yds. 1·6576 ft.; 8517·264 yds.; 1236 yds. 0 ft.; 96·768 in.; 19777·2288 yds.; 1 yd. 1 ft. 124·41 in.; 14079·68 yds.

[*N.B.* When this not very difficult problem of "discovering the area of an 8n from the side only" has been thoroughly studied, with the aid of the clue furnished by the subjoined diagram,—*then*, but not till then, let the following methods be examined.

a. Construct 8n as before, and complete the square (Note 4.) Then A M = D N.

Fig. 24.



Now ∠ A M E = 90°, and ∠ D A E = 135°.

∴ ∠ M A E = 180° − 135° = 45°. (Euc. I. 13.)

∴ ∠ M E A = 45°, and ∴ M A = M E. (Rt. ∠d △, (Note 5.)

$$\therefore \text{Area } \triangle \text{A M E} = \frac{\text{M A}^2}{2}.$$

∴ Area 4 corner △s = 2 M A² = A E² = A D². (*N.B.*)

Again: $\text{M A}^2 + \text{M E}^2$ or $2\,\text{M A}^2 = \text{A D}^2$ ∴ $\text{M A} = \sqrt{\frac{\text{A D}^2}{2}}$

$$\therefore \text{M A} + \text{D N} = 2\sqrt{\frac{\text{A D}^2}{2}} \quad \therefore \text{M N} = \text{A D} + 2\sqrt{\frac{\text{A D}^2}{2}}$$

$$\text{and } \therefore \text{M N}^2 = \left(\text{A D} + 2\sqrt{\frac{\text{A D}^2}{2}}\right)^2$$

Now the 8n = the square − the 4 corner △s

$$= \left(\text{A D} + 2\sqrt{\frac{\text{A D}^2}{2}}\right)^2 - \text{A D}^2$$

$$= 2\,\text{A D}^2 + 4\,\text{A D}\sqrt{\frac{\text{A D}^2}{2}} = \&c.$$

β. The same result may be obtained, without completing the square, thus: —

Having described the 8n join A G, D B, E K, F H, cutting each other at rt. ∠s.

Then O P R S is a square, and $= \text{A D}^2$; the 4 △s A O E, &c. $= 2\,\text{A O}^2 = \text{A D}^2$ (as before); and the 4 □s A O P D, &c. $= 4\,(\text{A D} \times \text{A O}) = 4\,\text{A D}\sqrt{\frac{\text{A D}^2}{2}}$;

$\therefore$ the whole 8n $= 2\,\text{A D}^2 + 4\,\text{A D}\sqrt{\frac{\text{A D}^2}{2}} = \&c.$

Case IV. Given, area of 8n: find side.

[*N.B.* The expression for area, found in *Case* III. may be modified thus:

area 8n $= 2\,\text{A D}^2 + 4\,\text{A D}^2\sqrt{\frac{1}{2}} = \text{A D}^2\,(2 + 4\sqrt{\cdot 5})$

$= \text{A D}^2 \times 2{\cdot}8944.$

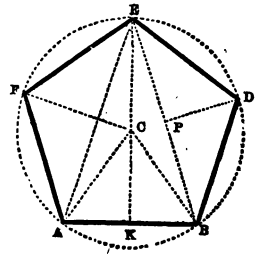
$\therefore \text{A D} = \sqrt{\frac{\text{area 8n}}{2{\cdot}8944}}.$

And this may be tried upon the 8ns whose area was determined in the last Case.]

THE PENTAGON.

1. The pentagon is treated of by Euclid, in Bk. IV. Props. 11—14., and employed in Prop. 16. It is involved also in Prop. 10. of the same Book as may be shown thus, (*vid.* Fig. in Potts' Euclid). Since the ∠s A B D and B D A each = twice ∠ B A D, $\therefore$ each ∠ $= \frac{2}{5}$ of 180°, *i.e.* 72°, and $\therefore$ ∠ B C D = 72° also. $\therefore$ ∠ A C D = 180° − 72° = 108°, the ∠ of a regular 5gon,

Fig. 25.



which accordingly would be described in the small ⊙, with a 10n in the larger one, if the figs. were completed.

2. The ∠ of a 5n being known, the fig. can be constructed as in Octagon, Note 3. β. And since it will contain 5 equal △s, its area is known (like any other reg. pol.) whenever a side, as A B, can be measured, and also C K the ⊥ upon it from the centre, *i. e.* the rad. of inscribed ⊙. (*Vid.* Reg. Pol., Note 4.)

Case I. Given, side of 5n and rad. of circumscribed ⊙; find area.

	Side.	*Radius.*	*Ans. Area.*
1.	60 yds.	51 yds.	6186·45 yds.
2.	20 yds.	17 yds.	687·38 yds.
3.	2 c. 40 lks.	2 c. 4 lks.	9 c. 8983½ lks.
4.	10 yds.	8 yds. 1 ft. 6 in.	171·845 yds.
5.	8½ yds.	7$\frac{1}{12}$ yds.	119 yds. 3 ft. 5¼ in.
6.	5 ft.	4¼ ft.	4 yds. 6 ft. 138·45 in.

Case II. Given, side and alternate diagonal, *i. e.* A B and E B: find area.

	Side.	*Diagonal.*	*Ans. Area.*
1.	75 yds.	122 yds.	9677 yds.
2.	4 ft. 2 in.	6 ft. 9 in.	3 yds. 2 ft. 125 in.
3.	150 p.	244 p.	241 a. 3 r. 28 p.
4.	2 c. 25 lks.	3 c. 66 lks.	8 c. 7093 lks.
5.	88 ft.	64 ft.	274·7535 yds.
6.	36 yds.	61 yds.	2215·658 yds.

[*N.B.* E B being known, D P and E K are both readily found, since E A = E B. Also △ A F E = △ B D E; and area of △ A E B is determined independently.]

Case III. Given, side and ⊥ upon it from ∠ opposite, *i. e.* A B and E K: find area.

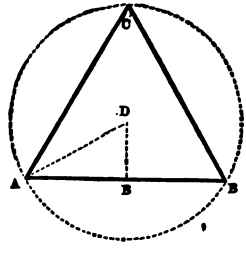
	Side.	*Perpendicular.*	*Ans. Area.*
1.	37 yds.	58 yds.	2355½ yds.
2.	25 ft.	38½ ft.	119 yds. 4 ft.
3.	21 yds.	32 yds.	759·244 yds.
4.	24 yds. 2 ft.	38 yds. 2 ft.	1046 yds. 8 ft.

	Side.	*Perpendicular.*	*Ans. Area.*
5.	50 yds.	77 yds.	4300 yds.
6.	111 yds.	174 yds.	4 a. 1 r. 20 p. 24½ yds.
7.	105 yds.	160 yds.	3 a. 3 r. 27 p. 14½ yds.
8.	31½ yds.	48 yds.	1708·3 yds.
9.	8 in.	12·31 in.	110 in.
10.	3 ft.	55·4 in.	1 yd. 6 ft. 69 in.
11.	40 ft.	61·55 ft.	305 yds. 7 ft.
12.	54 yds.	83 yds.	5017·5 yds.

THE CIRCLE.

1. Construct a trigon or equilat. △ A B C, of any convenient size; and from any one of its ∠s, as A, draw a straight line to its centre D. Now trace out a series of polns., from the square upwards, all having the same centre D, and all the same distance D A from this centre to each of their ∠s.

Fig. 26.



2. This being done, the student will make the following observations.

α. As the *number* of sides increases, their *length* diminishes, being at the maximum in the 3n.

β. As the number of ∠s increases, their size increases, being at the minimum in the 3n.

γ. As a direct consequence of this, each successive poln. encloses more space.

δ. In the 3n, 4n, and 5n, the side is *greater* than D A (which remains the same), the difference gradually lessening, until in the 6n they are equal; after which the side becomes *less* than D A, and in each succeeding poln. the difference is more and more perceptible.

ε. The ⊥ D E upon the side from the centre continually approximates to D A.

3. If now, under the same restrictions as before, the number of sides be increased indefinitely, their length will diminish in proportion, until they become mere *points* placed closely side by side. And with the perception of their *length* will also disappear all perception of the *angular space* which each adjacent pair encloses, so that the figure will in fact have lost (to the eye) the distinctive character of the *polygon* (γονυ) and melted into a *circle.*

4. Since no account can be taken of *sides* in the ⊙, as in any other polygon, it might be supposed that its circumference or perimeter can be found only by measuring its whole extent, or some definite portion of it, as $\frac{1}{2}$ or $\frac{1}{4}$. It is, however, possible to determine it very certainly without doing this, by *finding the length of its diameter*, or even its radius only, *i. e.* $\frac{1}{2}$ diameter, since it has been discovered that the ratio of circumference to diameter is as 3 : 1, nearly.

5. This is easy to bear in mind, and will serve well enough for rough estimates, where exactness is not required. But the student, who has gone carefully through the preceding polygons, will at once perceive its inaccuracy; since he will remember that this is exactly the ratio of a *hexagon's* perimeter to its bisecting diagonal, which of course is also the diameter of the ⊙ circumscribed about it. (Reg. Pol., Note 2.) And the perimeter of the latter must necessarily be greater than that of the figure which it encloses.

6. The true ratio, then, is 3·14159 &c. : 1, or $3\frac{1}{7}$: 1, very nearly, which latter is here adopted throughout, being easy to calculate and sufficiently correct for all ordinary practical purposes.

7. The utility of this method will be appreciated where the ⊙ under consideration is awkward to manage, from its size or other peculiarity, as a mill-wheel, steam paddle-wheel, &c. &c., which may be partly immersed in water,

hidden from view, or otherwise difficult of access or manipulation.

8. *E. g.* Suppose the circumference of one of these to be required, and only a spoke or rad. of the ⊙ within reach, whose length (allowing for axle and rim) is 7 feet. Then *diameter* is at once known to be 14 feet; and, remembering the ratio $3\frac{1}{7} : 1$, or $22 : 7$, the circumference is instantly seen to be 44 feet.

9. Had this wheel been a 6n instead of ⊙, its perimeter would have been just 6 times 7, or 42 feet. The student may satisfy himself as to the truth of the foregoing statements by actual measurement. Let him take a hoop, and roll it along the ground until it has gone round 7 times exactly; he will then find that the distance traversed is 22 times the hoop's diameter.

10. The circle follows the same law as other reg. polygons with regard to its area, which $= \frac{1}{2}$ perimeter × radius; for it will be noticed that the 2 lines D A and D E, which in other polygons unite centre and angle, and centre and side, respectively, coincide in the ⊙ and form one radius. (Note 2. ε.)

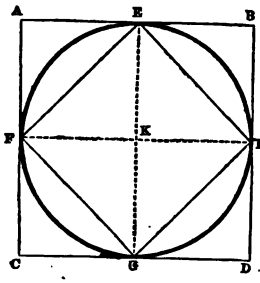
11. If, however, perimeter or circumference cannot be measured, it will be more convenient to adopt another form of the expression (for the area), *viz.* $\frac{3}{4}$ diameter^2 for rough estimates, or as before $\frac{3\frac{1}{7}}{4}$ diameter^2 for nicer calculations.

12. These two (in Notes 10. and 11.) are evidently identical; since in the former, perimeter $= 3\frac{1}{7}$ diameter, and rad. $= \frac{\text{diam.}}{2}$; while the latter may be otherwise written as $\left(\frac{3\frac{1}{7}\text{ diam.}}{2} \times \frac{\text{diam.}}{2}\right)$

13. This latter formula (viz. area ⊙ $= \frac{3\frac{1}{7}}{4}$ diameter^2) may be thus illustrated. Describe any square A B C D, and within it the square E F G H, which is seen to contain exactly $\frac{1}{2}$ A B C D. If now a ⊙ be described with centre K and rad.

K F, it will embrace the whole of the inner square, and *about* ½ the remaining space of the outer one; *i. e. about*

Fig. 27.



¾ of the whole square A B C D, whose *side* A B is the same as the *diameter* F H of the ⊙. It is seen ∴ to be geometrically true that area ⊙ $= \frac{3\frac{1}{7}}{4}$ diameter^2, or $\frac{11}{14}$ diameter^2.

14. A briefer form still, *viz.* area ⊙ $= 3\frac{1}{7}$ rad.^{2}, is obviously the same as before ("Square," Notes 11. and 12.) The former however, is recommended to the student, as more likely to retain its hold upon his memory, from his seeing the principle upon which it is founded, and the diagram given in illustration.

15. When it is said above, that the area of a polygon or circle is the "product of half the perimeter and the ⊥ upon it from the centre," it is meant that the area is the same as it would be if ½ perimeter were extended in a straight line, as the *length* of a rectangle or oblong, of which the ⊥ from centre forms the *breadth*. Finding this area is called *squaring* the figure. (*Vid. fig.*, Carpet and Paper.)

16. And since in the ⊙ the accurate determination of this square surface depends upon the length of the circumference and the relation which this bears to the diameter, the investigation of the problem called the "Quadrature or Squaring of the Circle," has much occupied the attention of mathematicians; the practical result being the discovery, that circumference : diam. (or semi-circumference : rad.) :: 3·14159 &c. : 1, or $3\frac{1}{7}$: 1, as above.

17. It is evident that the circumference and diameter of a ⊙ are in *direct proportion* to each other, *i. e.* if either of them be halved, doubled, trebled, &c., then the other is halved, doubled, trebled, &c. too, as a necessary consequence.

As in the reg. pol., however, (*vid.* Note 8.) so in the ⊙, "its area varies as the *square* of its diameter," *i.e.* if the diameter or circumference be halved, doubled, trebled, &c. the area becomes $\frac{1}{4}$, 4, 9, &c. times what it was before. The importance of this observation will be more clearly seen in the "Cylinder." (Euclid, XII. 2.)

18. The student will now understand how to determine what change must be made in the diameter or circumference of a ⊙ in order to effect any requisite increase or diminution of area.

Let A represent the area of that ⊙ whose diameter is D. Then, since if 4 A is wanted, we must have 2 D, *i. e.* $\sqrt{4}$ D;

if 9 A „ „ 3 D, *i. e.* $\sqrt{9}$ D;

if $\frac{1}{4}$ A „ „ $\frac{1}{2}$ D, *i. e.* $\sqrt{\frac{1}{4}}$ D;

&c. &c.

it follows, by the same law, that

if 2 A is wanted, we must have $\sqrt{2}$ D;

if 3 A „ „ $\sqrt{3}$ D;

if $\frac{1}{2}$ A „ „ $\sqrt{\frac{1}{2}}$ D.

&c. &c.

19. When the circumference only of a ⊙ is given, its centre may be found in a very easy and practical way by the method which Euclid suggests, Bk. III. Prop. 1., *viz.* "Draw any convenient *chord*, and then draw a straight line bisecting it at right angles, and meeting the circumference both ways. *The point of bisection of this straight line is the centre required.* (*Vid.* Sector, Note 2.)

Case I. Given diameter: find circumference and area.

[*N. B.* Circumference $= 3\frac{1}{7}$ diameter, and area $= \frac{3\frac{1}{7}}{4}$ diameter^2. *Vid.* Notes 6. and 11.]

	Diameter.	*Ans. Circumference.*	*Area.*
1.	77 yds.	242 yds.	4658$\frac{1}{2}$ yds.
2.	49 ft.	154 ft.	1886$\frac{1}{2}$ ft.
3.	19$\frac{1}{4}$ p.	60$\frac{1}{2}$ p.	1 a. 3 r. 11$\frac{5}{32}$ p.
4.	3 c. 85 lks.	12 c. 10 lks.	1 a. 1 c. 6462 lks.

	Diameter.	*Ans. Circumference.*	*Area.*
5.	32 yds. 2 ft.	102 yds. 2 ft.	27 p. 21 yds. $6\frac{1}{4}$ ft.
6.	12 yds. 2 ft. 6 in.	40 yds. 1 ft.	129 yds. 3 ft. 90 in.
7.	$115\frac{1}{2}$ p.	363 p.	65 a. 2 r. $1\frac{5}{8}$ p.
8.	11 c. 55 lks.	36 c. 30 lks.	10 a. 4 c. 8158 lks.

Case II. Given, circumference: find diameter and area. (*Vid. Case* I.)

	Circumference.	*Ans. Diameter.*	*Area.*
1.	132 yds.	42 yds.	1386 yds.
2.	198 yds.	63 yds.	$3118\frac{1}{2}$ yds.
3.	23 c. 76 lks.	7 c. 56 lks.	4 a. 4 c. 9064 lks.
4.	495 yds.	$157\frac{1}{2}$ yds.	4 a. 0 r. 4 p. $9\frac{5}{8}$ yds.
5.	396 yds.	126 yds.	2 a. 2 r. 12 p. 11 yds.
6.	14 c. 85 lks.	4 c. $72\frac{1}{2}$ lks.	1 a. 7 c. $5415\frac{5}{8}$ lks.
7.	15 ft. 7 in.	4 ft. $11\frac{1}{2}$ in.	2 yds. 2 ft. $50\frac{3}{8}$ in.
8.	91 yds. 2 ft.	29 yds. 0 ft. 6 in.	668 yds. 3 ft. 90 in.

[In Questions 1 and 5, observe Note 17.]

Case III. Given area: find diameter and circumference. (*Vid. Case* I.)

	Area.	*Ans. Diameter.*	*Circumference.*
1.	2464 ft.	18 yds. 2 ft.	58 yds. 2 ft.
2.	6237 yds.	89·09 yds.	279·997 yds.
3.	2 a. 0 r. 5 p. $24\frac{3}{4}$ yds.	112 yds.	352 yds.
4.	2772 yds.	59·397 yds.	186·676 yds.
5.	4 a. 4 c. 9064 lks.	8 c. 19 lks.	23 c. 76 lks.
6.	129 yds. 3 ft. 90 in.	12 yds. 2 ft. 6 in.	40 yds. 1 ft.
7.	10 a. 1 r. 9 p. $13\frac{3}{4}$ yds.	252 yds.	792 yds.
8.	4 a. 6 c. 5848 lks.	7 c. 70 lks.	24 c. 20 lks.

Case IV. (*Vide* Notes 17. and 18.)

1. The circles *ABD*, *EFG*, *HKN*, are "concentric" (*i. e.* described from same centre), and equidistant from each other. Radius *AC* being 28 ft., find whole area enclosed within each ring. (*Fig.* 28.)

Ans. 2464; 9856; 22176 ft.

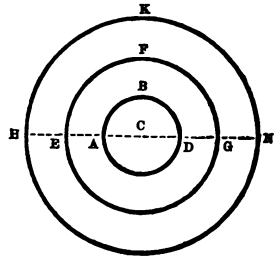
2. Find area of each of the two belts or zones. *Ans.* 7392; 12320 ft.

3. The circumferences of two circular playgrounds are 154 and 110 yds. Area of former is to be increased four times, and of the latter nine times. What will be the new diameter of each? *Ans.* 98 and 105 yds.

4. Diameter of a circle is $38\frac{1}{2}$ yds. How much more ground is enclosed by another whose diameter is 77 yds.? *Ans.* 3493[illegible] yds.

5. Circumference of a circle is 198 yds. How much less area is enclosed by another, whose circumference is $\frac{3}{4}$ the former? *Ans.* 1732½ yds.

Fig. 28.



6. Diameter of circle *HKN* being three times that of circle *ABD*, which is 42 yds.: find difference of area. *Ans.* 11088 yds.

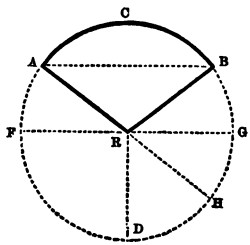
7. Two circular grass plots are 14 and 21 yds. diameter. The former is to be made thrice, and the latter twice, as large. What will be the new diameters? *Ans.* 24·248; 29·6982 yds.

8. If area of former be reduced $\frac{1}{3}$ (*i. e.* made only $\frac{1}{3}$ as large as before), and of the latter $\frac{1}{4}$ (*i. e.* made only $\frac{1}{4}$ as large as before), find new diameters. *Ans.* 8·0831; 9·39177 yds.

THE SECTOR.

1. "The sector of a circle is the figure contained by two straight lines drawn from the centre and the circumference between them." (Euclid, III. Def.)

Fig. 29.



2. In the annexed figure, R A C B is a sector, formed by the radii R A and R B, and A C B the "*arc*," (*i. e.* arch or bow); while A B is termed the "*chord*" of the arc (*i. e.* string of the bow), and the space A C B *cut off* between the two, a "segment."

3. The remaining portion of the circle, *viz.* R A D B, is a sector also.

4. The circle remaining the same, it will be seen that the size or area of the sector depends upon the length of the arc A C B, which again varies as the ∠ A R B at the centre; or *vice versâ*.

5. When ∠ A R B = 60°, (as ∠ D R H), the sector is called a "sextant," (60° being $\frac{1}{6}$ of 360°). When the radii are at right ∠s to each other, (as R D, R G,) they enclose an ∠ of 90°, and the sector becomes a "quadrant." When ∠ A R B = 180°, the radii form a diameter, as F G, with which the chord A B becomes coincident, and the sector then assumes the form of a segment.

6. The great importance of the circle will now be recognised in another point of view, *viz.* as furnishing the only means by which the size of *angles* can be determined. The circumference of every circle is regarded as if made up of 360 equal parts, called "degrees," being the number of daily *steps* by which the sun was anciently thought to accomplish his annual journey round the earth. And this mode of reckoning still holds, from its great convenience; the number 360 being peculiarly capable of subdivision into *aliquot parts, i. e.* without fractions. (*Vid.* Miscel. Qns. on Square, No. 72.)

7. The size of an angle, then, is estimated by reference to the arc which would subtend it, were it formed into a sector; and the degrees in the arc being counted, with the aid of a protractor, or otherwise, the angle is named accordingly. (*Vid.* Rt. ∠d △, Notes 2, &c.)

8. The sector being a portion of a circle, its area, &c., is estimated in the same way; *i. e.* area sector = $\frac{1}{2}$ perimeter × radius: the perimeter here being the *arc* which subtends the angle at centre. (*Vid.* Circle, Note 10.)

9. Trigonometry takes cognisance of angles *beyond* 180°; and in considering the sector, the student has them brought before his notice. For, as has been observed in Note 3., R A D B is a sector as well as R A C B; and accordingly the angular space at R must be taken account of *outside* the radii R A and R B, as well as *inside*, *i. e.* if inside ∠ A R B of sector R A C B = 120°, then outside ∠ A R B of sector R A D B = 360°—120° = 240°. (Euclid, I. 15.), and so on.

Case I. Given, arc and radius: find area. (*vid.* Note 8.)

	Arc.	*Radius.*	*Ans. Area.*
1.	42 yds.	21 yds.	441 sq. yds.
2.	3 ft. 2 in.	1 ft. 7 in.	2 ft. 73 in.
3.	143 yds.	47 yds. 2 ft.	3408 yds. 1 ft. 72 in.
4.	16 ft. 6 in.	2 ft. 11 in.	2 yds. 6 ft. 9 in.
5.	177 yds.	28 yds.	Impossible. Why?
6.	37½ p.	9½ p.	1 a. 0 r. 15 p. nearly.
7.	13 c. 75 lks.	8 c. 68 lks.	5 a. 9 c. 6750 lks.
8.	19 c. 53 lks.	6 c. 51 lks.	6 a. 3 c. 5701 lks.

9. When does the Sector become a Segment?

Case II. Given, angle at centre, *i. e.* number of degrees in arc, and radius: find area.

[*N.B.* Arc : circumference :: degrees in arc : 360°.]

	Angle at centre.	*Radius.*	*Ans. Area.*
1.	45°	2 ft. 11 in.	3 ft. 49¼ in.
2.	60°	18 yds. 2 ft.	102 yds. 4 ft. 96 in.
3.	72°	5 ft. 3 in.	1 yd. 8 ft. 46 in.
4.	105°	4 ft. 8 in.	2 yds. 1 ft. 138½ in.
5.	120°	8·75 p.	2 r. 0 p. 6$\frac{23}{36}$ yds.
6.	150°	5 c. 25 lks.	3 a. 6 c. 0937 lks.
7.	270°	9⅛ yds.	226·38 yds.
8.	225°	11⅔ in.	1 ft. 123$\frac{13}{36}$ in.
9.	240°	21 yds.	924 yds.
10.	315°	56 yds.	8624 yds.

Case III. Given, radius and angle at centre: find length of arc. (*Vid. Case* II.)

	Radius.	*Angle at centre.*	*Ans. Arc.*
1.	10½ in.	60°	11 in.
2.	31½ yds.	90°	49½ yds.
3.	56 ft.	180°	58 yds. 2 ft.
4.	5 ft. 3 in.	225°	6 yds. 2 ft. 7½ in.
5.	22¾ yds.	300°	119⅛ yds.
6.	8 yds. 2 ft. 3 in.	240°	36⅔ yds.

Case IV. Given, area and radius: find arc and angle at centre. (*Vid. Case* II.)

[*N.B.* $\frac{\text{arc}}{2} \times \text{rad.} = \text{area}$; $\therefore$ arc = &c., and, degrees in arc : 360° :: arc : circumference $\therefore$ $\angle$ at centre $= \frac{360° \text{ arc}}{\text{circum.}}$.]

	Area.	*Radius.*	*Ans. Arc.*	*∠ at centre.*
1.	882 yds.	42 yds.	42 yds.	$57\frac{3}{11}°$
2.	2 yds. 6 ft. 9 in.	35 in.	16 ft. 6 in.	324°
3.	5 ft. 2 in.	3 ft. 2 in.	38 in.	$57·27\dot{2}7\dot{2}°$
4.	$1642\frac{3}{4}$ ft.	18 yds. 2 ft.	19 yds. 1 ft. 8 in.	60°
5.	80 yds. 0 ft. $7\frac{1}{2}$ in.	$8\frac{3}{4}$ yds.	55 ft.	120°
6.	226·38 ft.	9·8 ft.	46 ft. $2\frac{2}{3}$ in.	270°

Case V. Given, area and arc: find radius and angle at centre. (*Vid. Case* III.)

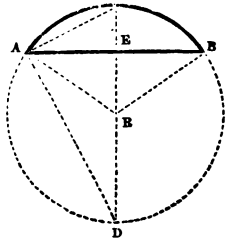
	Area.	*Arc.*	*Ans. Radius.*	*∠ at centre.*
1.	4928 ft.	58 yds. 2 ft.	56 ft.	180°
2.	$481\frac{1}{4}$ yds.	36 yds. 2 ft.	8 yds. 2 ft. 3 in.	240°
3.	1764 yds.	84 yds.	42 yds.	$114\frac{6}{11}°$
4.	150·92 ft.	15·4 ft.	6 yds. 1 ft. $7\frac{1}{2}$ in.	45°
5.	8 ft. 3 in.	5·5 ft.	35 in.	108°
6.	$15\frac{1}{2}$ ft.	9 ft. 6 in.	38 in.	$171\frac{9}{11}°$

7. Draw diagrams describing sectors with ∠ at centre, respectively 45°, 60°, 90°, 180°, 225°, 240°, 300°, 315°, 125°. Radius being of any convenient length.

THE SEGMENT.

1. A segment of a circle is the figure contained by a straight line and the circumference which it cuts off. (Euclid. III., Def. and Prop. 25. &c.)

Fig. 30.



2. The upper portion of the span of a bridge is usually of this figure.

3. To find area of segment A C B, area of sector R A C B must first be discovered, and area △ R A B deducted from it; the remainder is the area required.

4. When the angle at centre is 180° or more, △ R A B is of course *not* deducted.

Case I. Given, arc, chord, and rad.: find area. (Note 3.)

	Arc.	*Chord.*	*Radius.*	*Ans. Area.*
1.	88 yds.	$72\frac{3}{4}$ yds.	42 yds.	1084·167 yds.
2.	$39\frac{1}{3}$ yds.	$32\frac{1}{3}$ yds.	84 yds.	310 yds.
3.	264 yds.	70 yds.	56 yds.	1 a. 2 r. 2 p. 11 yds.
4.	$39\frac{1}{3}$ yds.	35 yds.	28 yds.	162 yds. 5 ft.
5.	539 yds.	$59\frac{1}{2}$ yds.	98 yds.	5 a. 1 r. 3 p. $2\frac{3}{4}$ yds.
6.	$117\frac{1}{3}$ yds.	$48\frac{1}{2}$ yds.	28 yds.	31 p. $25\frac{11}{12}$ yds.

7. When is the segment a sector too?

Case II. Given, chord, radius, and degrees in arc: find area. (*Vid.* Sector.)

	Chord.	*Radius.*	∠ *at centre.*	*Ans. Area.*
1.	$48\frac{1}{2}$ yds.	28 yds.	120°	$481\frac{5}{6}$ yds.
2.	70 yds.	35 yds.	180°	1 r. 23 p. $19\frac{1}{4}$ yds.
3.	$23\frac{1}{3}$ yds.	$18\frac{3}{5}$ yds.	90°	504 yds.
4.	28 yds.	28 yds.	60°	71·185 yds.
5.	60·62 yds.	35 yds.	240°	2 r. 4 p. $25\frac{3}{5}$ yds.
6.	$9\frac{1}{3}$ yds.	$9\frac{1}{3}$ yds.	300°	265·8683 yds.

Case III. Given, chord, degrees in arc, and ⊥ ht. at centre: find arc and area.

	Chord.	*Degrees in Arc.*	⊥ *ht.*	*Arc.*	*Area.*
1.	43 yds.	90°	9 yds.	47·426 yds.	260·3 yds.
2.	56 yds.	135°	19 yds.	71·14 yds.	760·44 yds.
3.	86 yds.	270°	104 yds.	284·56 yds.	8587·85 yds.
4.	112 yds.	225°	84 yds.	237·132 yds.	7156·54 yds.
5.	29 yds.	80°	5 yds.	32·8 yds.	118 yds.
6.	$41\frac{1}{2}$ yds.	120°	12 yds.	50·3 yds.	354·4 yds.
7.	45 yds.	140°	$16\frac{1}{2}$ yds.	$57\frac{2}{3}$ yds.	521 yds.

[*N. B.* For the solution of the above, the length of E R (*vid. fig.* Segment) is required, in order to determine C R, the radius. Should the student be unable, after various trials, to invent a method of his own for finding it, the following may assist him. Since C A D is a semicircle (Euc. III. 1.), ∴ ∠ D A C = 90°. (Euc. III. 31.) And ∵ A E is drawn from ∠ D A C ⊥ to D C ∴ it is a "mean proportional" between D E and E C, (Euc. VI. 8. Cor. and 13.), *i. e.* D E :

A E :: A E : E C, $\therefore$ D E $\times$ E C = A E^2, (since "Product of extremes = Product of means"); $\therefore$ D E $= \frac{\text{A E}^2}{\text{E C}}$ $\therefore$ D E + E C (*i. e.* D C) $= \frac{\text{A E}^2}{\text{E C}} + \text{E C}$ $\therefore$ C R $= \frac{1}{2}\left(\frac{\text{A E}^2}{\text{E C}} + \text{E C}\right)$.

Or thus, more briefly, from Euc. III. 35.,

D E $\times$ E C = A E $\times$ E B = A E^2; $\therefore$ &c. &c.]

Case IV. Given, chord of arc, and height at centre: find radius of circle.

	Chord.	*Height.*	*Ans. Radius.*
1.	76 yds.	21 yds.	$44\frac{37}{42}$ yds.
2.	25 yds.	4 yds.	$21\frac{17}{32}$ yds.
3.	60 yds.	11 yds.	$46\frac{2}{22}$ yds.
4.	157 yds.	49 yds.	$87\frac{148}{392}$ yds.
5.	44 yds.	18 yds.	$22\frac{4}{9}$ yds.
6.	40 yds.	31 yds.	$21\frac{40}{93}$ yds.

[*N. B.* C E and A E being known, E R may be found by constructing a sector only, thus:

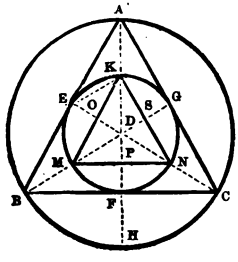
C R or A R = C E + E R $\therefore$ (C E + E R)^2 = A E^2 + E R^2

$\therefore$ E R $= \frac{\text{A E}^2 - \text{C E}^2}{2 \cdot \text{C E}}$ $\therefore$ C R $= \frac{1}{2}\left(\frac{\text{A E}^2}{\text{E C}} + \text{E C}\right)$ as before.]

RELATION OF THE CIRCLE TO TRIGON AND TETRAGON.

1. It has been shown (Circle, Note 13.) that the circle occupies $\frac{31}{4}$ or $\frac{11}{14}$ of the area of its circumscribed 4n. And since the inner square is just half the outer one, the circle must be $\frac{22}{14}$ of its inscribed 4n.

Fig. 31.



2. It is important, too, to know the relation which exists between the circle and its circumscribed and inscribed 3ns, and to remember

(α) that diam. $\odot$: $\perp$ ht. of circumscribed 3n :: 2 : 3,
(β) that diam. $\odot$: $\perp$ ht. of inscribed 3n :: 4 : 3.

3. It may be shown thus:

Construct reg. 3n A B C, (Hexagon, Notes 3, 4.) Bisect the $\angle$s and draw lines meeting in centre D. (Euc. IV. 4.) These being produced will also bisect the sides at rt. $\angle$s in E, F, G. (Euc. I. 9.)

Describe the circle E F G. Then by joining K, E, M, &c., a reg. 6n is formed, $\therefore$ D E K is a reg. 3n. $\therefore$ K E = K D, and $\angle$ K E D = 60°.

It follows $\therefore$ that $\angle$ K E A = 30° and since $\angle$ B A C is bisected by A F, $\therefore$ $\angle$ K A E = 30° also; $\therefore$ A K = (K E =) K D = D F $\therefore$ K F = $\frac{2}{3}$ A F,

i. e. (α) K F : A F :: 2 : 3.

Now describe circle about A B C; then F H = A K. $\therefore$ A H : A F :: 4 : 3. And since the inscribed 3n K M N is exactly similar to A B C, in this case also it is true that

(β) K F : K P :: 4 : 3.

4. This may also be shown arithmetically, thus:—

Let B C = 16 yds.; then A F = 13·8564 yds.;

$\therefore$ area $\triangle$ A B C = 110·8512 sq. yds.

Now since $\triangle$ D B F = $\frac{1}{8}$ $\triangle$ A B C,

$$\therefore \frac{\text{D F} \times \text{B F}}{2} = 18{\cdot}4752 \text{ sq. yds.}$$

$\therefore$ D F = 4·6188 yds. = $\frac{1}{3}$ of 13·8564 yds.

$\therefore$ K F = $\frac{2}{3}$ A F, as before.

5. The student may satisfy himself by actual calculation that the areas of the outer $\odot$ and 3n are quadruple those of the inner $\odot$ and 3n respectively. (*Vid.* Reg. Pol. Note 8.)

Case I. Given, side of inscribed $\triangle$: find area of circle.

16 yds.; 24 yds.; 28 in.; 40 ft.

Answers.

268·1728 yds.; 603·3888 yds.; 5 ft. 101·28 in.; 186 yds. 2 ft.

Case II. Given, side of circumscribed triangle: find area of circle.

14 yds.; 21 yds.; 35 ft.; 3 ft. 6 in.

Answers.

$51\frac{1}{3}$ yds.; $115\frac{1}{2}$ yds.; 35 yds. $5\frac{5}{8}$ ft.; $3\frac{5}{24}$ ft.

Case III. Given, side of inscribed triangle: find area of segment M N F.

20 in.; 25 yds.; 35 ft.; 24 yds.

Answers.

82 in.; $128\frac{1}{4}$ yds.; 27 yds. $8\frac{1}{8}$ ft.; 118·08 yds.

Case IV. Given, side of circumscribed triangle: find area A E K G.

16 ft.; 20 yds.; 45 in.; 30 yds.

Answers.

14·6015 ft.; 22·8148 yds.; $115\frac{1}{2}$ in.; $51\frac{1}{3}$ yds.

Case V. Given, diameter of circle; find area of inscribed and circumscribed triangles.

(1) 14 ft.; (2) 28 yds.; (3) $17\frac{1}{2}$ in.; (4) 10 ft. 8 in.; (5) 18 yds.; (6) 2 ft. 3 in.

Answers.

(1) 7·0723 and 28·2892 yds.; (2) 254·604 yds. and 1018·416 yds.; (3) 11·05 yds. and 44·203 yds.; (4) 4·105 yds. and 16·42 yds.; (5) $105\frac{1}{4}$ yds. and 421 yds.; (6) 1·644 ft. and 6·576 ft.

[*N. B.* The student will find his advantage in puzzling out for himself the foregoing cases, which will not prove difficult when studied with the aid of the notes and diagram.]

Case VI. Given, ⊥ height of trigon; find by how much its area exceeds and falls short of the areas respectively of its inscribed and circumscribed ⊙s.

(1) 16 ft.; (2) 20 in.; (3) 21 yds.; (4) 14 ft.; (5) 3 c. 84 lks.; (6) 35 p.

Answers.

(1) $58\frac{2}{3}$ ft. and $209\frac{1}{3}$ ft.; (2) $90\frac{1}{8}$ in. and $327\frac{2}{7}$ in.; (3) 100·598 yds. and 361·397 yds.; (4) 44·71 yds. and 160·622 yds.; (5) 3 c. 3641 lks. and 12 c. 836 lks.; (6) 279·467 p. and 1003·866 p.

Case VII. (Note 1.)

1. Side of a square is 42 yds. Find area of inscribed and circumscribed circles. *Ans.* 1386 and 2772 yds.

2. Area of a square is 1 yd. 7 ft. 97 in. Find diameter of inscribed and circumscribed circles. *Ans.* 4 ft. 1 in.; 69·926 in.

3. Area of a circle is 1 yd. 8 ft. 16 in. Find side of inscribed square. *Ans.* 39·598 in.

4. Circumference of a circle is 2 ft. 9 in. Find perimeter of circumscribed square. *Ans.* 42 in.

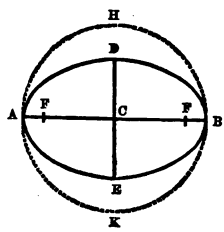
5. Diameter of circle is 35 ft. Find area of circumscribed square. *Ans.* 1225 ft.

6. Diagonal of square is 54 yds. Find area of inscribed circle. *Ans.* 1145¼ yds.

THE ELLIPSE.

1. The ellipse A D B E (ἐκ λείπω, "leave out") may be considered as a circle with a portion *left out* or omitted, the left out figures A D B H and A E B K being called "Lunes," from their crescent or lunar character.

Fig. 32.



2. The ellipse derives an especial importance and interest from the fact, that this is the figure described by the earth and all the other planets in their journey round the sun.

3. All straight lines passing through centre C are of course *diameters;* but the greatest and least are A B and D E (which bisect each other at rt. ∠s in C), and are therefore called the *major* and *minor* diameters or "axes."

4. With D or E as centre and C A or C B as radius describe a ⊙ cutting the major axis in F, F. These 2 points are the "Foci," and are so situated that A F = F B and F C = C F.

5. If two pins be fixed as foci at any convenient distance from each other, and a thread with its ends tied be

thrown loosely over them both, then by moving a pencil round within the thread (which must be kept upon the stretch) a perfect ellipse will be traced out.

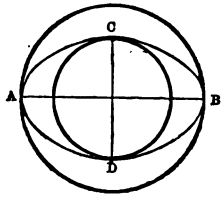
6. This mode of construction renders obvious a remarkable property of the ellipse, viz., that "to whatever point in the curve two straight lines are drawn from the foci, *their sum is always the same*, being always equal to the major axis."

7. The ellipse is more or less elliptical or *eccentric*, in proportion as the foci are farther from or nearer to the centre. When they coincide with A and B, the ellipse becomes a *straight line*; when with C, a *circle*.

8. The ellipse is a *mean proportional* (*vid.* Appendix) between its inscribed and circumscribed circles, *i. e.* the ⊙s described about its minor and major axes as diameters.

e. g. Let areas of major and minor ⊙s be 16 yds. and 9 yds., and let x represent area of ellipse.

Fig. 33.



Then, $16 : x :: x : 9$

$\therefore x^2 = 16 \times 9$, and $\therefore x = 12$ yds.

9. The ellipse, being closely allied to the circle, follows the same laws of calculation.

e. g. Circumference of ellipse = $3\frac{1}{7}$ mean diameter.

And area of do. $= \frac{3\frac{1}{7}}{4}$ (mean diameter)^2

10. "Mean diameter" is the *medium* or *average* of the major and minor diameters, and $\therefore = \frac{AB + CD}{2}$. (*Vid.* Questions on Square, No. 32. Note).

Case I. Given, major and minor axes: find Perimeter. (Note 9.)

Major diameter.	*Minor diameter.*	*Ans. Circumference.*
1. 35 yds.	21 yds.	88 yds.
2. 53 p.	31 p.	132 p.
3. 12 c. 74 lks.	9 c. 94 lks.	35 c. 64 lks.

Major diameter.	*Minor diameter.*	*Ans. Circumference*
4. 27 c. 58 lks.	19 c. 74 lks.	74 c. 36 lks.
5. $31\frac{1}{2}$ ft.	$22\frac{3}{4}$ ft.	28 yds. 1 ft. 3 in.
6. $17\frac{1}{2}$ in.	$12\frac{1}{4}$ in.	3 ft. $10\frac{3}{4}$ in.

Case II. Given, perimeter and one axis: find the other.

Circumference.	*Axis* 1.	*Ans. Axis* 2.
1. 7 ft. 4 in.	1 ft. 9 in.	2 ft. 11 in.
2. 44 yds.	31 ft.	17 yds. 2 ft.
3. 15 yds. 1 ft. 9 in.	$12\frac{1}{4}$ ft.	5 yds. 2 ft. 6 in.
4. $85\frac{1}{4}$ p.	$31\frac{1}{2}$ p.	$22\frac{3}{4}$ in.

[*N.B.* Let x represent either of the two diameters, and y the other. Then (Note 9.)

$$\frac{x+y}{2} \times 3\tfrac{1}{7} = \bigcirc\text{ce, or } 11x + 11y = 7 \bigcirc\text{ce.}$$

$$\therefore x = \frac{7 \bigcirc\text{ce} - 11y}{11}.]$$

Case III. Given, perimeter and proportion of the two diameters: to find them.

[*e. g.* $\bigcirc$ce = 66 and diameters are in the ∷ n

2 : 1, *i. e.* $\frac{2}{3}$ and $\frac{1}{3}$.

Now $x + y = \frac{7}{11} \bigcirc$ce (*Vid. Case* II.)

$= \frac{7}{11} \times 66 = 42.$

$\therefore x = \frac{2}{3}$ of 42, and $y = \frac{1}{3}$ of 42.

i. e., $x = 28$, and $y = 14$.]

Circumference.	*Proportion of diameters.*	*Diameters.*
1. 29 yds. 1 ft.	5 : 3	35 ft. and 21 ft.
2. 7 ft. 4 in.	4 : 3	32 in. and 24 in.
3. 143 yds.	8 : 5	56 yds. and 35 yds.
4. 11 ft.	5 : 9	30 in. and 54 in.

Case IV. Given, major and minor axes; find area. (Note 9.)

Major axis.	*Minor axis.*	*Ans. Area.*
1. 48 yds.	35 yds.	1320 yds.
2. 29 in.	21 in.	$478\frac{1}{2}$ in.
3. 7 ft. 9 in.	4 ft. 8 in.	3 yds. 1 ft. 60 in.
4. $19\frac{1}{2}$ yds.	$13\frac{1}{2}$ yds.	209 yds. 3 ft. $77\frac{1}{4}$ in.

	Major axis.	*Minor axis.*	*Ans. Area.*
5.	$13\frac{1}{2}$ p.	$7\frac{3}{4}$ p.	82 p. 6 yds. 2 ft.
6.	12 c. 85 lks.	8 c. 75 lks.	8 a. 8 c. 3437 lks.
7.	17 c. 92 lks.	11 c. 29 lks.	15 a. 8 c. 9632 lks.
8.	5 p. $3\frac{1}{2}$ yds.	2 p. $4\frac{1}{2}$ yds.	377 yds. $4\frac{3}{4}$ ft.

[*N.B.* Area $= \frac{3\frac{1}{7}}{4}$ (major axis $\times$ minor axis).

$= \frac{11}{14}$ (mean diameter)^2]

Case V. Given, area and one axis; find the other. (Note 9.)

	Area.	*Diameter A.*	*Ans. Diameter B.*
1.	3 ft. $46\frac{1}{2}$ in.	2 ft. 5 in.	1 ft. 9 in.
2.	1 r. 3 p. $19\frac{1}{4}$ yds.	48 yds.	35 yds.
3.	3 yds. 1 ft. 60 in.	4 ft. 8 in.	7 ft. 9 in.
4.	17 a. 6 c. 6875 lks.	17 c. 50 lks.	12 c. 85 lks.
5.	1 a. 0 r. 4 p. 12 yds. 4 ft.	$15\frac{1}{2}$ p.	$13\frac{1}{2}$ p.
6.	418 yds. 7 ft. $10\frac{2}{7}$ in.	$26\frac{1}{4}$ yds.	$19\frac{1}{4}$ yds.

[*N.B.* Axis A $\times$ axis B $\frac{3\frac{1}{7}}{4}$ = area (Note 9.)

$$\therefore \text{Axis A} = \frac{\text{Area}}{\text{Axis B}} \times \frac{4}{3\frac{1}{7}}.]$$

Case VI. Given, areas of circumscribed and inscribed circles: find area of ellipse.

[*N.B.* 1. Find the mean proportional between the two areas (Note 8.), or

2. Find the diameters of the two circles, *i. e.* the major and minor axes of the ellipse, and then proceed as in *Case* IV.

Either method will give area of ellipse.]

	Area circumscribed circle.	*Area inscribed circle.*	*Area, Ellipse.*
1.	2464 yds.	1386 yds.	1848 yds.
2.	616 yds.	273 yds. 7 ft.	410 yds. 6 ft.
3.	1 yd. 4 ft. $14\frac{1}{2}$ in.	4 ft. 40 in.	7 ft. 70 in.
4.	$962\frac{1}{2}$ yds.	$346\frac{1}{2}$ yds.	$577\frac{1}{2}$ yds.

THE FRAME OR RING.

1. This is of any figure, according to the nature and shape of the enclosed area: *e. g.* it may be a rectangular

Fig. 34.

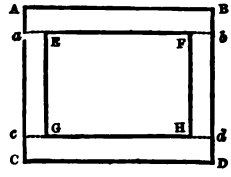
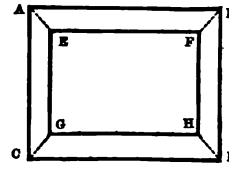


Fig. 35.



picture frame to be gilt, a pathway round a circular or elliptical lawn to be gravelled, &c.

2. Its area is readily determined by finding (1) area of whole space enclosed by external perimeter A B C D, (2) area of E F G H, and subtracting latter from former.

3. Or thus, find the *mean perimeter*, and multiply this by the ring's breadth.

e. g. External dimensions of an oblong frame are (28 × 21) inches, and internal do. (22 × 15) inches, *i. e.* frame is 3 inches wide.

α. Area = (28 × 21) − (22 × 15) in. = 258 in.

β. Area = $\frac{98 + 74}{2} \times 3 = 258$ in.

γ. Area = 2 A *b* + 2 *a* G = 168 + 90 = 258 in.

4. The method suggested in γ can be employed only when the frame is rectangular.

5 "Mean perimeter" = ½ (ext. ○ce + int. ○ce); *i. e.* add the *outside* perimeter to *inside* do. and take half this sum as the medium or average. (*Vid.* Note 3. γ.)

6. That it is the "mean perimeter" which must be employed, will be evident to the student if he considers what figure will be produced by cutting open the zone or belt (as in *fig.* 36.) and stretching it out (as in *fig.* 37.). The result is a *trapezoid*, whose area cannot of course be correctly obtained by finding the product of either (A A × C D) or (B B × C D), the former being as much too small as the latter is too large.

Fig. 36.

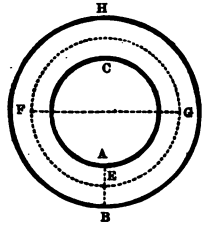
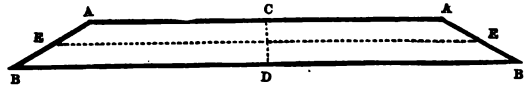


Fig. 37.



7. The dotted circle E F G is the "mean perimeter" and F G the mean diameter.

8. The mean perimeter and diameter of the ellipse are found in the same way.

9. The sides of an ordinary picture frame (as in *fig.* 35.) form 4 trapezoids, equal two and two.

10. The student may advantageously employ the different methods suggested in Note 3. to check and prove each other.

11. It must not be forgotten that if one side of a picture, for example, measures 20 inches, and a 2-inch frame be added, its new length will be, not 22, but 24 inches, since frame's width is added to *both ends* of the side.

1. Dimensions of a picture-frame are (39 × 27) inches externally, and (34 × 22) inches internally. Find surface to be gilded. *Ans.* 2 ft. 17 in.

2. Length of a rectangular field is 547 yds., and breadth 389 yds. A road 10 ft. wide runs along every side of it *outside* the hedge, and another of the same width *inside* do. Find area to be gravelled in each case. *Ans.* 6284 yds. 4 ft.; 6195 yds. 5 ft.

3. A circular fish-pond of 35 ft. diameter is to have a turf edging 3½ ft. wide. Dimensions of each turf are (21 × 14) inches. How many will be wanted? *Ans.* 207¾.

4. A circular table measures 5 yds. 2 ft. 5 in. in extreme circumference.

It is formed of a circular slab of alabaster surrounded by a band of black marble 1½ inches wide. How many sq. ft. of marble are there?

Ans. 12½.

5. The longest and shortest diameters of an elliptical lawn are 94½ and 47 yds.; and the carriage drive round it is 17½ ft. wide. Find area to be gravelled. *Ans.* 1587 yds. 3½ ft.

6. The major and minor axes of an elliptic picture-frame are 38½ and 25 inches (extreme measurements), and its breadth is 3½ inches. Find surface to be gilded. *Ans.* 2 ft. 39½ in.

7. Area of a square picture is 5 ft. 64 in. with the frame, and 4 ft. 49 in. without it. Find width of frame. *Ans.* 1½ inches.

8. A rectangular window, 5½ ft. wide including the border, has a semicircular head springing from it at a height of 11 ft. from the sill or base. Its border, which goes all round, consists of stained glass 14 inches wide. Find its cost at 2*s.* 6*d.* a sq. ft. *Ans.* £4 11*s.* 4½*d.*

9. A mirror is (20 × 18) inches, including frame, which is ⅛ of the whole area. Find its width. *Ans.* 1 inch.

10. A riding-school (60 × 40) yds. has stabling built uniformly on every side, and at such a distance from it that the area of the whole yard exactly equals that of the school. Find its width. *Ans.* 10 yds.

11. In a rectangular lawn is constructed a fish-pond with an uniform breadth of turf on every side. The lawn is 200 ft. long, and 60 yds. wide, and pond occupies ¼ whole area. Find its dimensions and width of turf. *Ans.* (120 × 100) ft.; 40 ft.

12. The canvas of a painting is (20 × 16) inches; and its area is just ⅘ of what the area of whole picture will be when frame is added. Find breadth of frame. *Ans.* 2 inches.

MISCELLANEOUS QUESTIONS ON THE POLYGONS, CIRCLE, ELLIPSE, ETC.

[*N.B.* Draw diagrams, as before.]

1. Diameter of circular space *A* is 49 yds.
   Do. *B* is 98 yds.
   Find circumference and area of each. *Ans.* { 154 & 1886½ yds. / 308 & 7546 yds.

2. Circumference of do. *A* is 132 yds.
   Do. *B* is 264 yds.
   Find diameter and area of each. *Ans.* { 42 & 1386 yds. / 84 & 5544 yds.

3. Area do. A is 2464 yds.
Do. B is 9856 yds.
Find diameter and circumference of each. *Ans.* { 56 & 176 yds. / 112 & 352 yds.

4. Show that "$180° x - 360°$" is a general expression for the sum of all the interior angles of any regular polygon of x sides. (*Vid.* Reg. Polygon, Notes.)

5. Hence determine the value of an interior angle in a 10n, 12n, 15n, respectively.

6. The tire of a wheel is $3\frac{1}{2}$ inches broad, while a spoke is 19 inches. Find area of tire. *Ans.* 418 inches.

7. A mill-sail is 21 ft. long, and revolves regularly 9 times in a minute. Find distance traversed by its extremity in $1\frac{1}{4}$ hours.
Ans. 16 m. 1540 yds.

8. A square court has a circular green inscribed in it, radius $45\frac{1}{2}$ ft. How much of the court is not grass? *Ans.* 15 yds. 8·798 ft.

9. A circular meadow containing $6506\frac{1}{2}$ sq. rods is bisected by a fence which costs 2*s.* 3*d.* a yd. Find cost of this fence, and also of one going all round. *Ans.* £56 6*s.* $1\frac{1}{2}$*d.*; £176 19*s.* 3*d.*

10. Area of a circular green, cut out of a sq. court, and touching each side, is 616 yds. Find cost of fencing it round at 1*s.* 3*d.* a ft., and space left for gravel in each corner. *Ans.* £16 10*s.*; 42 yds.

11. The axes of an elliptical garden are $13\frac{1}{2}$ p. and $7\frac{3}{4}$ p. Find its value at £90 15*s.* per acre. *Ans.* £46 12*s.* $6\frac{1}{4}$*d.*

12. Find the sum (1) of *all* the exterior angles of a regular 5n, 6n, 8n, 10n; (2.) of *one* of each. *Ans.* 1440°; 213°.

13. The interior angles of a regular 6n are in arithmetical progression, the largest being to the smallest as 28 : 8. Find them.
Ans. Largest is $186\frac{2}{3}°$ &c.

14. Compare the areas of a 6n and 8n, whose sides are 7 ft. and 6 ft. respectively, the side of the internal square in latter being 11 ft.
Ans. { 6n, 127·302 ft. / 8n, 173·7538 ft.

15. The side of a hexagonal field is 247 yds. Find value at £80 13*s.* 4*d.* per acre. *Ans.* £2639 11*s.* $9\frac{9}{16}$*d.*

16. A walk $1\frac{3}{4}$ yds. wide is made round the outside of a circular bowling green, 189 ft. diameter. Find expense of gravelling it at 2*s.* 3*d.* per dozen sq. yds. *Ans.* £3 6*s.* $9\frac{8}{33}$*d.*

17. A carriage drive $5\frac{1}{2}$ yds. in width encompasses an elliptic lawn whose axes are 12 c. 74 lks. and 9 c. 94 lks. Find area of drive and of lawn.
Ans. { Drive, 9 c. $1064\frac{1}{2}$ lks. / Lawn, 10 a. 1 c. 394 lks.

18. The drive being fenced on each side at 2*s.* 6*d.* a yd., how much will the cost of the *outer* perimeter exceed that of the *inner*?
Ans. £2 3*s.* $2\frac{1}{2}$*d.*

19. A barn is 35 ft. long and 26 ft. wide. Roof projects 2 ft. beyond its *spring*, from which to ridge is 17 ft. perpendicular height. Find cost of slates (18 × 13) inches, at 57*s.* a gross. *Ans.* 19 guineas.

20. A gable is 27 ft. wide, and 19 ft. perpendicular from eave to ridge. Find cost of weather-boarding the two sloping edges at 7*s.* 6*d.* a yd.

Ans. £5 16*s.* 6¾*d.*

21. A circular bed, 4 ft. diameter, contains 37 flower-pots. What is the circumference of another which holds 148, and of a third which holds 222 pots of the same size? *Ans.* 25¼ ft.; 30·787 ft.

22. (A.) A circular lawn, diameter 84 yds., is divided into 2 equal portions by a circular fence which follows the curve traced out by a shorter radius moving about the same centre. Find radius.

Ans. 29·7 yds.

22. (B.) Prove by actual calculation that this radius *will* give half the area; and show that if the lawn be encircled by a belt of shrubbery containing exactly the same space, its width is 17·3964 yds.

23. The spokes or radii of the hind and fore wheels of a coach are as 4 : 3, and the smaller turns round 14080 times in 21 miles. Find radius of each. *Ans.* 1 ft. 9 in.; 1 ft. 3¾ in.

24. Side of a 5n is 38 ft., and alternate diagonal 64 ft.
Do. 8n is 16 yds., and bisecting diagonal 42 yds.
Do. 6n is 18 yds. Find area of each.

Ans. { 274·7535 yds.; 1242 yds.; 841 yds.

25. A semicircular headed window is 3½ ft. wide and 13 ft. extreme height. Find cost of glazing it at 24*s.* a sq. yd. *Ans.* £5 17*s.* 10*d.*

26. Three fields, containing 1 a. 3 r. 35 p., 2 a. 1 r. 38 p., and 3 a. 2 r. 23 p., are exchanged for a square meadow of same area which needs fencing. Find cost at 4*s.* 9¾*d.* a rod. *Ans.* £6 18*s.* 4*d.*

27. The diagonal and side of a square room measure 48·284 ft., and the side is a whole number. Find it. *Ans.* 20 ft.

28. The major axis of an ellipse is 48 yds.; and if its minor axis were 7 yds. shorter, its area would be 1056 yds. Find area. *Ans.* 1320 yds.

29. The minor axis of an ellipse is 4 ft. 8 in., and its area, 3 yds. 1 ft. 60 in. Find area of each of the *left out* lunes. *Ans.* 1 yd. 0 ft. 55$\frac{29}{32}$ in.

30. A slate is 4 inches longer than wide, and its half-inch frame contains $\frac{1}{18}$ sq. ft. Find its dimensions. *Ans.* 15 and 11 inches.

31. At £60 10*s.* per acre, a circular meadow is worth £69 6*s.* Find its diameter. *Ans.* 84 yds.

32. Compare the areas of a 5n, 6n, and 8n, a side of each being 36 yds. and the diagonal of 5n 61 yds.

Ans. { *P.* 2215·658; *H.* 3367; ⊙ 6330 yds.

33. A circular pond is 17 ft. 6 in. in diameter, and has a walk 3 ft. 6 in. wide all round. What is the walk's area? *Ans.* 25¾ yds.

34. At 7*s.* 6¾*d.* per sq. rod, an elliptic plot of ground fetches £16 10*s.* One axis is 35 yds. Find the other. *Ans.* 48 yds.

35. A piece of land (100 × 78) yds. is encompassed by a ditch which occupies 1340 sq. yds. What will be its dimensions when the ditch is filled up? *Ans.* 110 & 88 yds.

36. Carpet being 3*s.* 7½*d.* a yd., 29 inches wide, find cost of flooring an octagonal room whose side is 12 ft. *Ans.* £17 7*s.* 7½*d.*

37. An oblong flower-bed, containing 144 sq. ft. is inscribed in a circle of 30 ft. diameter. Find its sides. *Ans.* 24 & 6 ft.

38. A circular pond has area 346½ yds. Find cost of fencing at 1*s.* 6*d.* a yd. *Ans.* £4 19*s.*

39. The interior angles of a triangle are as 3, 4, 5, and the shortest side is 8 yds. Find sides and angles.

Ans. 24, 32, 40 ft.; 105°, 120°, 135°.

40. Diameter of circular grass plot A is 14 ft. more than that of B, and area of $B = \frac{1}{3} A$. Find circumference of each.

Ans. 88 and 132 ft.

41. A gable end is 13¾ yds. wide, 15½ yds. from floor to eave, and 5 yds. perpendicular from eave to ridge. Find number of bricks (9 × 3) inches required for its facing. *Ans.* 11808.

42. The areas of the circles described about the major and minor axes of an ellipse are 1296 and 576 yds. Find area of ellipse.

Ans. 864 yds.

43. Determine the length of the two axes in the foregoing ellipse.

Ans. { 40·6134; 27·0756.

44. Find expense of fencing one of the lunes at 8*s.* 3*d.* a rod.

Ans. £1 19*s.* 1*d.*

45. The major axis of an ellipse exceeds the minor by 20 yds., and its area is 1584 yds. Find them. *Ans.* 56 and 36 yds.

46. Find the perimeter of a square inscribed in one half of a circle whose diameter is 16 yds. *Ans.* 28·6 yds.

47. In the rectangular space fronting a house, and measuring (20 × 14) yds. a semicircular grass plot, radius 10½ yds., is laid down with turfs (18 × 10½) inches. Find cost of turfs at 4*s.* 2*d.* a score, and of gravelling the rest at 3*s.* 4½*d.* a sq. yd. *Ans.* £12 7*s.* 6*d.*; £18 0*s.* 3⅜*d.*

48. What different areas will a 16 yd. cord enclose when arranged in the form of (1.) square, (2.) circle, (3.) equilat. triangle, (4.) Isosc. triangle with base 4 yds. *Ans.* 16; 20¼; 12·3168; 11·312 yds.

49. A grass plot, regular 3n, with side 12 yds., has a circular fish-pond dug in its centre touching each side. Find area of pond, and grass left

Ans. 47½; 24·7 yds.

50. Find area of a circular segment, arc 22 yds., chord 18 yds. and radius 30 yds. *Ans.* 72·438 yds.

51. Floor with carpet, 6*s.* 9*d.* a yd., 27 inches wide, an octagonal room, side 4 yds., and a 6n, side 10 ft. *Ans.* £33 5*s.* 3*d.*; £12 19*s.* 9½*d.*

52. A circular space, containing 55594 sq. rods, is bisected by a fence which costs 1*s.* 9*d.* a yd. Find value of (1.) ground at 3*d.* a yd., (2.) fence across, (3.) fence all round at 3*s.* 6*d.* a yd.

Ans. £21021 9*s.* 3¾*d.*; £128 0*s.* 3*d.*; £804 12*s.* 11*d.*

53. How many medals 2¼ inches diameter can be struck out of a sq. sheet of metal, one side of which is 15¾ ft.? And how much metal will then be left? *Ans.* 7056; 5 yds. 8 ft. 22½ inches.

54. In a rectangular space (20 × 12) yds., a semicircular grass plot is formed with radius $1\frac{3}{11}$ pole. Find cost of laying it with turfs (18 × 12) inches, worth 4*s.* 7*d.* a score, and of gravelling the rest at 1*s.* 8½*d.* sq. yd.

Ans. £5 5*s.* 10½*d.*; £13 15*s.* 0½*d.*

55. Area of a circular bowling green, cut out of an equilateral triangular garden, and touching each side, is 2464 yds. Find circumference and space left in corner. *Ans.* 176 yds.; 536·6 yds.

56. Areas of the circles described about the minor and major axes of an ellipse are 43 and 89 yds. Find area of ellipse.

Ans. 61·86275 yds.

57. Find cost of painting 2 tables at ¾*d.* a sq. inch. *A* being an 8n, side 3 ft., and *B* a 6n, side 4 ft. *Ans.* £19 11*s.* 1½*d.*; £18 14*s.* 1½*d.*

58. *A*, *B*, *C*, run same way round a circular field, diameter 63 yds., at rates of 40, 50, 60 yds. per half minute. They start together. When will they be together again? *Ans.* 9′ 54″.

59. A window is a regular 3n, height 10 ft., with a circle inscribed. Glass for the circle costs 4*s.* 6*d.* per sq. ft., and for the rest 2*s.* 6*d.* do. Find cost of window. *Ans.* £10 14*s.* 2*d.*

60. $\frac{3}{14}$ of a square, being the portion not enclosed by the inscribed circle, = 42 yds. Find perimeter of square and circle. *Ans.* 56 and 44 yds.

61. The radius of a semicircle is 17 ft. 6 in. Find area.

Ans. 481½ ft.

62. A circular barrack-yard contains an acre. How often will a wheel, 1¾ ft. radius, turn in going round it? *Ans.* 67½ times.

63. A triangular meadow has its sides 126, 198, 254 yds. A straight ditch runs across it between the points of bisection of the 2 longest sides. Determine length of ditch and area of the 2 portions of meadow. (Euclid. VI. 2.)

Ans. { 63 yds. / 3062·224 yds. / 9186·672 yds.

64. How many coins, ⅝ inch diameter, can be struck from a plate of metal ⅝ yd. square. And how much will be left? *Ans.* 1296; 108 in.

65. A ladder, 40 ft. long, stands upright against a house. How far must its foot be pulled away so as to lower the top 1 yd.?

Ans. 15·198 ft.

66. The radii of 2 concentric circles are 5 and $7\frac{1}{2}$ ft. Find area of the zone formed by these rings. *Ans.* 10 yds. 8·175 ft.

67. The area of a circle is 11 yds. 1 ft. Find area of squares circumscribed and inscribed. *Ans.* 28 yds. 2·64 ft.; 63·66 ft.

68. Perimeter of a regular 3n is 45·588 ft. Find perimeter of a square containing an equal area. *Ans.* 40 ft.

69. A ladder 37 ft. long, being planted 8 ft. from the base of a house, reaches $\frac{3}{4}$ of whole height. How much longer must it be to reach the top? *Ans.* 17·773 ft.

70. ABC is a triangle right-angled at B, which is distant 4 yds. from A, and 16 ft. from C. A line DE running parallel to BC cuts off 24 sq. ft. Find sides of this triangle. (Eucl. VI. 2.)

Ans. 6, 8, 10 ft.

71. Two clerks (A, paid yearly; B, half yearly), will each receive a salary of £100 for the first year: after which they are to be raised, viz. A, £10 every Christmas, and B £5 every 6 months. What difference will there be in their total receipts after 5 years' payments?

Ans. B £80 more than A.

72. Expense of fencing round a circular pond at 1*s.* 6*d.* a yd. is £4 19*s.* Find area. *Ans.* $346\frac{1}{2}$ yds.

73. The inscribed square is cut out of a circle, radius 4 yds. Find area left. *Ans.* $18\frac{2}{7}$ yds.

74. A rectangular fish-pond measures 36 ft. one way, and 54 ft. the other, and has an uniform breadth of turf on every side, whose area just equals that of pond. Find its breadth. *Ans.* 9 ft.

75. A courtyard is a regular 3n, and its paving at 6*s.* a sq. yd. cost just twice as much as walling it all round at 3*s.* 6*d.* a ft. Find side.

Ans. 24 yds.

76. What figure is formed by bisecting the sides of a trapezium, and joining the adjacent points of bisection? *Ans.* A $\square$ or parallelogram.

77. Show that if any two sides of any triangle be bisected, and the points of bisection joined, the line of junction = $\frac{1}{2}$ the third side, and area of new triangle = $\frac{1}{4}$ original.

78. A sq. yd. is divided into 4 equal squares by lines bisecting the sides at right angles. Find space remaining after the 4 inscribed circles are cut out. *Ans.* $1\frac{13}{14}$ ft.

79. Light takes 8 minutes to travel from the sun to the earth, moving at rate of 200,000 miles per second. Find the length of the earth's orbit or annual journey round the sun. *Ans.* 600 million miles.

80. A triangular field has its vertical angle bisected by a drain which runs straight across and divides the opposite ditch into 2 portions of 72

and 58 yds. The sum of the other 2 sides = 270 yds. Find value of field at £60 10s. per acre. (Eucl. VI. Prop. B.) *Ans.* £93 16s.

81. One sq. inch being allowed for the area of a safety-valve (circular) in a steam-engine to every 15 sq. ft. of boiler surface, find diameter of the valve to a boiler whose surface contains 1000 sq. ft. *Ans.* 9·21 inches.

82. A single valve is required to replace 3 smaller ones containing severally 10, 20, 30, sq. inches. Find its diameter. *Ans.* 8·738 inches.

83. A boiler needs a safety-valve of 10 inches diameter, but has only an 8-inch one. Find diameter of the second valve which is applied to make up the deficiency. *Ans.* 6 inches.

84. The load on a safety-valve is 500 lbs., steam pressure being 10 lbs. per sq. inch. Find diameter of valve. *Ans.* 8 inches, nearly.

85. Radius of valve *A* is 1 inch less than radius of *B*, and their areas together = area of *C*, whose radius is 5 inches. Find rad. of *A* and *B*. *Ans.* 3 and 4 inches.

86. Diameter of circular riding school is 56 yds. Find width of stable yard which encompasses it, and whose area is $\frac{1}{4}$ that of school. *Ans.* $3\frac{1}{10}$ yds.

87. (A.) A cow is tethered so that she can graze a circular area of $\frac{1}{4}$ rood. Find length of tether. *Ans.* $9\frac{1}{2}$ yds.

87. (B) Allowing 7 ft. as the cow's length from horns to tail, how much more surface may she graze if the rope be tied to her tail instead of to her horns? *Ans.* $160\frac{83}{100}$ yds.

88. A measuring wheel is $3\frac{1}{2}$ ft. diameter, and turns exactly 168 times in going round a circular pond. Find area. *Ans.* 30184 yds.

89. How must a circular plot of ground, 343 ft. diameter, be laid out, so that a fish-pond in the centre, a flower-garden round the pond, and a walk round the garden, may each occupy $\frac{1}{3}$ of the whole area.

Ans. { Pond's diameter, 198 ft. nearly.
Width of garden, 41 ft. Walk, $31\frac{1}{2}$ ft.

90. Value of an elliptic meadow, at 7*s.* $6\frac{3}{4}$*d.* a rod, is £26 16*s.* 3*d.*; and major axis is 65 yds. Find cost of fencing at 6*s.* 5*d.* a rod. *Ans.* £9 16*s.* 2*d.*

91. Cost of railing a circular lawn at 1*s.* 6*d.* a yd. is £6 12*s.* Find its value at £80 13*s.* 4*d.* an acre. *Ans.* £10 5*s.* 4*d.*

92. A circular piece of ground, 189 yds. diameter, is divided by concentric circles into 3 portions, whose areas are as 7, 5, 3, the central one being the largest. With what length of radius must each be described? *Ans.* 64·5557; 84·5233; $94\frac{1}{2}$ yds.

93. The lunes, formed by cutting an ellipse out of a circle whose diameter is 7 ft. 9 in., together contain $2703\frac{9}{11}$ inches. Find minor axis. *Ans.* 4 ft. 8 in.

94. A number of persons sit down to a round table, each being al-

lowed 22 inches, which arc subtends an angle of 30° at centre. The table is covered with round dishes, 33 inches in circumference, ranged side by side in concentric circles. Determine (1.) number of sitters, (2.) number of dishes, (3.) tabular space left unoccupied.

Ans. 12; 49; 9 ft. $3\frac{3}{8}$ in.

95. A viaduct consists of 27 semicircular arches springing from piers 13 ft. high, while the arches are 10 ft. 6 inches perpendicular height between chord and crown. They are all to be completely enclosed for warehouses, with bricks on one side and boarding on the other. Find cost of material, plank being $4\frac{3}{4}d.$ a sq. ft., and bricks $15s.\ 7\frac{1}{2}d.$ per 1000.

Ans. { Boards, £238 9*s.* $3\frac{9}{16}d.$ / Bricks, £50 4*s.* $0\frac{3}{4}d.$

96. A horse-breaking ring, 84 ft. diameter, is rolled with a roller 7 ft. wide, and a yard in diameter, which, starting from any point in the ring's circumference, passes round along its edge, and thence forms a series of continually diminishing circles until it reaches the centre. Find number of revolutions made, and total distance travelled by the roller in thus once traversing the whole ring. [*N.B.* Find mean circumference of the 6 concentric zones.] *Ans.* $85\frac{1}{2}$; 803 ft.

PART II.

MENSURATION OF SOLIDS.

[*Obs.* Parts II. and III. occupy a smaller portion of the book than Part I., not from their being of less importance, but from the circumstance that cubic measurement is little more than the *application of plane mensuration to the sides, &c. of solid bodies*; and considerable space having been devoted to the establishment and elucidation of the principles upon which the latter rests, it has not been thought needful to dwell at such length upon the former.]

1. SUPERFICIES or surface has only two dimensions, viz. length and breadth; solidity has three, viz. length, breadth, and thickness or depth.

2. Take a block of stone, wood, or other uniform substance, measuring 3 feet along each edge. This is a perfect cube, and forms a cubic or solid yard; each face being a square yard.

3. To determine the number of cubic feet in it.

On any of its six faces, line out the 9 square feet contained in it, and imagine these lines carried perpendicularly down through the block to its base. It will then be divided into nine equal bars, each 3 feet long and 1 foot square at the ends. Every one of these bars may then be divided into 3 blocks, each 1 foot long and 1 foot square at the ends; and the whole number of them will be 9 times 3, or 27.

4. Or thus: Imagine the block cut into 3 layers or slices, each 1 foot thick. Each of these layers may then be lined out as before, and cut through; when the same number of exactly equal cubes will be produced; viz. 9 in each slice, and 3 times 9, or 27, in the whole.

5. In the same way, a cubic foot may be divided into

1728 cubic inches. Hence, to determine the *cubic content* of any solid, shaped as a brick, book, &c.; "Find the area of any one face, and multiply this by the other dimension," or briefly, "find the product of its 3 dimensions."

6. Every solid such as the above is called a *Rectangular Parallelopipèd* (παραλληλος ἐπι πέδον); each of the 6 faces being equal and parallel to its opposite, and bounded by straight lines at right angles to each other. (Euc. XI. 6. &c.)

7. When the parallelopipèd is not rectangular, but rhomboidal, its solid content is still correctly obtained by finding the area of any face, and multiplying it by the perpendicular distance between this face and its opposite; since the solidity of the particles composing the body (as *a pack of cards*, for example), must always be the same in whatever way they may be arranged out of the perpendicular, *i. e.* whether they form a mass of rectangular or rhomboidal character.

8. Similarly for any mass having only 2 ends parallel. Obtain the area of one end, whether it be a triangle, parallelogram, trapezium, polygon, circle, &c., and then find the product of this area, and its perpendicular distance from the other end. This product is its solidity.

9. When the mass is of irregular shape (as a haystack, &c.), it must be divided as nearly as possible into regular portions, and the content of these ascertained separately. Then the content of the whole will be the sum of the contents of the several portions.

10. It must be borne in mind that, as there are 3 dimensions in a solid, the content of any mass increases or diminishes in the proportion of the *cube* of that number which increases or diminishes each side. (Euc. XII. 33.)

11. Thus, if the dimensions of a cistern are $(4 \times 3 \times 2)$ ft., it will contain 24 cubic ft., or about 150 gals.

Now, if *one* side only (as *length*, for example), be doubled, this content becomes simply *twice* as large, since dimensions become $(8 \times 3 \times 2)$, or $(4 \times 3 \times 2) \times 2$, *i. e.* 24×2, which = 48 cubic ft., or 300 gals.

If *two* sides (as length and breadth) be doubled, content is increased *four* times, since dimensions become $(8\times6\times2)$, or $(4\times3\times2)\times4$, *i. e.* 24×2^2, which $=96$ ft., or 600 gals.

But if *all three* sides be doubled (*i. e.* altered in the proportion of the same no. 2.) then content is increased *eight* times (*i. e.* as the *cube* of 2); since dimensions become $(8\times6\times4)$, or $(4\times3\times2)\times8$, *i. e.* 24×2^3, which $=$ 192 cubic ft., or 1200 gals.

And so of any other increase or diminution in the same way.

12. If it be required to increase or diminish all the 3 dimensions of a solid *proportionally*, so as to effect any requisite proportional increase or diminution in its cubic content, the proper method of doing so may be apprehended by consulting Plane Mensuration, "Oblong," Note 16.: from which, by parity of reasoning, it will be seen, that if the solid is to be made twice as large, without altering the relative proportion of the sides, each dimension must be multiplied by $\sqrt[3]{2}$; if *thrice*, by $\sqrt[3]{3}$, and so on.

CUBIC MEASURE, OR MEASURES OF SOLIDITY.

1728 cubic inches (*i. e.* 12 × 12 × 12) = 1 cubic foot.
27 cubic feet (*i. e.* 3 × 3 × 3) = 1 cubic yard.

1 cubic foot of water weighs 1000 oz., or 62½ lbs.
1 gallon ditto ditto 10 lbs. nearly.

1 gallon contains 277 cubic inches.

1 cubic inch of iron weighs 4⅓ oz., avoid.
1 ditto lead 6⅔ oz., ditto.
1 ditto powder ½ oz., ditto.

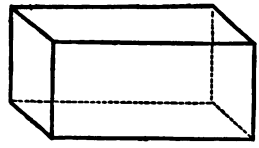
2 pints = 1 quart; 4 quarts = 1 gallon.
2 gallons = 1 peck; 4 pecks = 1 bushel.

The student will notice the alternation of the nos. 2 and 4 in this last couplet. And he may easily contrive to master all the above statements, as he is not asked to burden his memory with any facts but such as are likely to be of real use to him.

THE PARALLELOPIPED.

This is the simplest form of the solid, having (as a brick, book, &c.) 6 plane sides or faces, all parallelograms, of which the opposites are equal and parallel, two and two.

Fig. 38.



Case I. Given, length, breadth and depth or thickness: find solidity.

[*N.B.* Solidity = product of 3 dimensions. *Vid.* Note 5.]

	Length.	*Breadth.*	*Depth.*
1.	19 in.	17 in.	15 in.
2.	$9\frac{3}{4}$ ft.	$7\frac{1}{2}$ ft.	$6\frac{1}{4}$ ft.
3.	$9\frac{1}{3}$ ft.	$8\frac{5}{6}$ yds.	$7\frac{3}{4}$ ft.
4.	$12\frac{5}{8}$ yds.	$9\frac{5}{8}$ yds.	$7\frac{7}{11}$ yds.
5.	$17\frac{3}{4}$ yds.	$13\frac{1}{8}$ yds.	$11\frac{1}{3}$ yds.
6.	$4\frac{1}{2}$ ft.	$1\frac{1}{2}$ yd.	54 in.
7.	7 ft. 9 in.	6 ft. 8 in.	5 ft. 4 in.
8.	639 in.	$17\frac{3}{4}$ yds.	53 ft. 3 in.
9.	4 ft. 2 in.	4 yds. 1 ft.	256 ft.
10.	$37\frac{1}{2}$ yds.	$29\frac{3}{4}$ yds.	$13\frac{1}{4}$ yds.
11.	2 yds. 2 ft. 8 in.	2 yds. 1 ft. 7 in.	1 yd. 2 ft. 9 in.
12.	15 yds. 1 ft. 9 in.	13 yds. 2 ft. 5 in.	9 yds. 2 ft. 11 in.

Ans. Solidity.

1. 2 ft. 1389 in.
2. 16 yds. 25 ft. 54 in.
3. 24 yds. 13 ft. 1320 in.
4. 936 yds. 7 ft. 72 in.
5. 2726 yds. 16 ft. 216 in.
6. 3 yds. 10 ft. 216 in.
7. 10 yds. 5 ft. 960 in.
8. 5592 yds. 9 ft. 1215 in.
9. 1725 yds. 17 ft.
10. 236512½ yds.
11. 13 yds. 26 ft. 1560 in.
12. 2141 yds. 10 ft. 908 in.

Case II. Given, solidity and 2 dimensions: find third.

[*N.B.* Solidity $= L \times B \times D$ $\therefore L = \frac{S}{B \times D}$ &c.]

						Ans.	
1.	S.	2156 yds.	B.	14 yds.	D.	22 yds.	L. 7 yds.
2.	S.	135 yds. 3 ft.	L.	8 yds.	B.	6 yds. 1 ft.	D. 8 ft.
3.	S.	41 yds. 9 ft. 486 in.	D.	$6\frac{3}{4}$ ft.	L.	$10\frac{1}{2}$ ft.	B. $5\frac{1}{4}$ yds.
4.	S.	3293 yds. 9 ft.	B.	13 ft.	D.	19 ft.	L. 120 yds.
5.	S.	4 yds. 16 ft. 54 in.	L.	$15\frac{3}{4}$ ft.	B.	$2\frac{1}{4}$ ft.	D. $3\frac{1}{2}$ ft.
6.	S.	165 yds. 19 ft. 216 in.	D.	$3\frac{1}{2}$ yds.	L.	$10\frac{1}{2}$ yds.	B. $13\frac{1}{4}$ ft.

Case III. Given, solidity of a perfect cube: find length of edge.

85184 yds. 681472 ft. 3 yds. 26 ft. 297 in.
9 yds. 1 ft. 243 in. 11645 yds. 17 ft. 23557 yds. 17 ft.
10 yds. 15 ft. 559 in. 592704 yds.

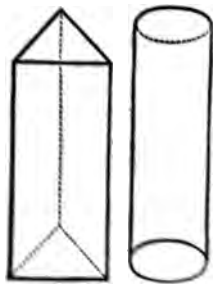
Answers.

44 yds.	29 yds. 1 ft.	4 ft. 9 in.	$2\frac{1}{13}$ yds.
22 yds. 2 ft.	28 yds. 2 ft.	6 ft. 7 in.	84 yds.

THE PRISM AND CYLINDER.

1. A Prism (πρίω, to saw) is a solid whose 2 *ends* or bases are similar, equal, and parallel plane surfaces, but may be of any conceivable shape or figure; while all its *sides* or faces are parallelograms, (being, as it were, *sawn* even;) except of course in the case of prisms standing on circular or other curvilinear bases.

Figs. 39 & 40.



2. When the prism has a △ for its end or base, it is called a triangular prism: when a 5n, a pentagonal prism when a square or oblong, a rectangular prism; and so on, according to the figure of its base. When the base is a circle, the prism becomes a *cylinder*.

3. When the prism is made of white glass, or other transparent material, it becomes singularly useful in the analysis of sunlight, which, in passing through the prism (usually triangular) is "refracted" or *broken up* into its component parts, and found to consist of "violet, indigo, blue, green, yellow, orange, and red," in this order. These are accordingly termed the "seven prismatic colours."

4. Solidity of prism = area of end × perpendicular height.

Find solidity of the following prisms: —

1. Square base, side 13 inches; height 19 inches. *Ans.* 1 ft. 1483 in.
2. Square base, diagonal 2 yds. 2 ft.; height 7 yds. 2 ft. *Ans.* 27 yds. 7 ft.
3. Oblong base ($3\frac{1}{2} \times 2\frac{1}{4}$) ft.; height $15\frac{3}{4}$ ft. *Ans.* 4 yds. 16 ft. 54 in.
4. Oblong base. Side $4\frac{1}{2}$ yds., diagonal $7\frac{1}{2}$ yds.; height 18 yds. *Ans.* 486 yds.
5. Triangle. Base $5\frac{1}{2}$ ft., $\perp$ $7\frac{1}{2}$ ft.; height 5 yds. 1 ft. *Ans.* 24 yds. 12 ft.
6. Right $\angle$d $\triangle$. Side 15 inches, hypothenuse 25 in.; height $17\frac{1}{2}$ ft. *Ans.* 18 ft. 396 in.
7. Regular 3n. Side $1\frac{3}{5}$ yd.; height 23 ft. *Ans.* 9 yds. 6 ft. 12 in.
8. Isosceles $\triangle$. Side 5 ft. 5 in.; base 8 ft. 8 in.; height $16\frac{3}{4}$ ft. *Ans.* 8 yds. 19 ft. 1548 in.
9. Scalene $\triangle$. Sides 5 ft. 9 in., 7 ft. 8 in., 9 ft. 7 in.; height $15\frac{5}{8}$ yds. *Ans.* 38 yds. 20 ft. 1692 in.
10. Trapezium. Diagonal 13 yds.; 2 $\perp$s $29\frac{1}{2}$ and $36\frac{1}{2}$ ft.; height $23\frac{3}{4}$ yds. *Ans.* 3396 yds. 2 ft. 36 in.
11. Trapezium. Diagonal 2 c. 48 lks.; 2 $\perp$s 13 c. 70 lks. and 9 c. 60 lks.; height 22 yds. *Ans.* 28 c. 892000 lks.
12. Trapezoid. 2 //s $26\frac{1}{2}$ and $19\frac{1}{2}$ yds.; $\perp$ 62 yds.; height $13\frac{1}{2}$ p. *Ans.* $105880\frac{1}{2}$ yds.
13. Trapezoid. 2 //s 23 and 17 in.; $\perp$ 4 ft. 8 in.; height 13 ft. 6 in. *Ans.* 15 ft.
14. Diamond. 2 diagonals 11 and 9 ft.; height $3\frac{1}{2}$ p. *Ans.* $105\frac{7}{8}$ yds.
15. Diamond. Diagonal 4 ft. 8 in.; side 2 ft. 11 in.; height $7\frac{1}{2}$ yds. *Ans.* 6 yds. 21 ft. 1296 in.
16. Hexagon. Side 4 ft. 9 in.; height 8 yds. 1 ft. *Ans.* 54 yds. 7 ft. 810 in.
17. Hexagon. Bisecting diagonal 3 ft. 2 in.; height 5 ft. 5 in. *Ans.* 1 yd. 8 ft. $483\frac{1}{4}$ in.

18. Octagon. Side 6 inches; height 4 ft. 8 in. *Ans.* 5 ft. 1090 in.
19. Octagon. Side 3 ft. 11 in.; height 45 yds. *Ans.* 360 yds. 10 ft. 216 in.
20. Pentagon. Side 3 ft., radius inscribed ⊙ 25 inches; height $13\frac{1}{3}$ yds. *Ans.* 23 yds. 24 ft. 1440 in.
21. Pentagon. Side 8 ft. 9 in., radius circumscribed ⊙ $7\frac{1}{2}$ ft.; height 16 yds. $2\frac{3}{4}$ ft. *Ans.* 250 yds. 12 ft. 1365 in.
22. Pentagon. Side 24 yds. 2 ft., alternate diagonal 40 yds. 2 ft.; height $5\frac{1}{2}$ p. *Ans.* 10545 yds. 7 ft.
23. Pentagon. Side 37 inches, ⊥ from angle to opposite side 58 in.; height $13\frac{1}{2}$ ft. *Ans.* 8 yds. 4 ft. 1112 in.
24. Decagon. Side 19 inches, radius inscribed ⊙ 2 ft. 5 in.; height 56 inches. *Ans.* 3 yds. 8 ft. 488 in.
25. Dodecagon. Side 5 ft. 2 in., radius circumscribed ⊙ 10 ft.; height 3 yds. 2 ft. *Ans.* 121 yds. 26 ft. $849\frac{3}{4}$ in.
26. Circle. Radius $38\frac{1}{2}$ inches; height 17 ft. 6 in. *Ans.* 20 yds. 26 ft. 237 in.
27. Circle, ⊙ce 12 ft. 10 in.; height $16\frac{1}{2}$ yds. *Ans.* 23 yds. 21 ft. 1205 in.
28. Sector. Arc $16\frac{1}{2}$ ft., radius 35 in.; height 17 ft. 9 in. *Ans.* 15 yds. 22 ft. 189 in.
29. Segment. Arc $29\frac{1}{4}$ yds., chord $24\frac{1}{4}$ yds., radius 14 yds.; height $125\frac{3}{4}$ yds. *Ans.* $15148\frac{1}{4}$ yds.
30. Ellipse. Major and minor axes $17\frac{1}{2}$ and $10\frac{1}{2}$ in.; height 5 ft. 4 in. *Ans.* 5 ft. 600 in.

THE WEDGE.

Fig. 41.

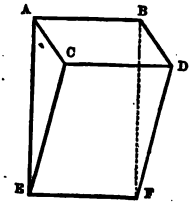


Fig. 42.

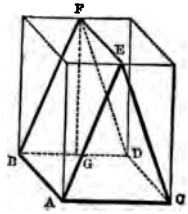
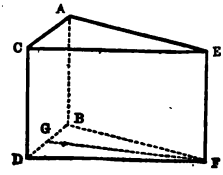


Fig. 43.



Case I. When the *edge* or thin end E F equals A B or C D, *the depth* of the *back*, the wedge is a triangular prism, D B F

and C A E being the equal and // ends or bases. Solidity of wedge then = area D B F × E F. In this, the common wedge, D B F and C A E are isosc. △s, while the other 3 sides are rectangular ▭s, A B F E and C D F E being similar and equal. *Obs.* This wedge is half the rectangular parallelopiped which would stand on A B C D as base, with G F as ⌊ height.

Find solidity of wedge in following instances (*vid. fig.*):—

1. E F is 5 in.; B D 4 in.; F G 9½ in. *Ans.* 95 in.
2. E F 7 in.; B D 5½ in.; F G 13½ in. *Ans.* 259⅞ in.
3. A B 13 in.; B F 23 in.; A C 9 in. *Ans.* 1319½ in.
4. A C 11 ft.; F D 21 ft.; C D 11 ft. *Ans.* 1226$\frac{3}{10}$ ft.
5. G F 17½ yds.; C A 7½ yds.; F E 15 yds. *Ans.* 984$\frac{3}{8}$ yds.
6. D B 18½ yds.; C D 19½ yds.; C E 29 yds. *Ans.* 4957·6 yds.

Case II. When the *edge* is either less or greater than the *back* in depth, then solidity = area B D F × mean depth.

[N. B. "Mean depth" is found by adding together the 3//s A B, C D, E F, and taking ⅓ sum. *Obs.* The *back* is supposed to be a ▭.]

Find solidity of wedge in following examples:—

1. A B 5 ft. 10 in.; E F 9 ft. 2 in.; B D 2½ ft.; F G 24½ in. *Ans.* 17 ft. 1624 in.
2. A B 10 yds. 2 ft.; E F 7 yds.; A C 1½ yds.; F G 4 yds. 2 ft. *Ans.* 892½ ft.
3. E F 6 in.; A B 9 in.; B D 13 in.; B F 19 in. *Ans.* 928·382 in.
4. A B 22½ yds.; E F 27 yds.; B D 15 yds.; F D 23 yds. *Ans.* 3913·7 yds.
5. C D 17½ ft.; F E 43 ft.; G F 17½ yds.; A C 13½ yds. *Ans.* 3071¼ yds.
6. A B 19½ in.; A C 13 in.; E F 15 in.; B F 25 in. *Ans.* 1 ft. 1096½ in.

THE PYRAMID AND CONE.

1. The Pyramid may stand on a base of any figure. Its sides or walls taper off gradually until they meet in a point at the vertex or summit, and thus form a series of triangular faces, all equal when the base is a regular polygon.

2. The pyramid thus loses both length and breadth at the summit, *i. e.* 2 out of its 3 dimensions: and the student may hence keep in mind the fact that "the solidity of any pyra-

Fig. 44.

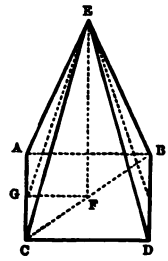
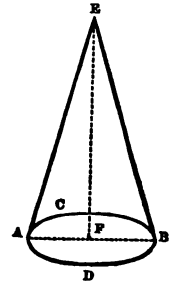


Fig. 45.



mid is just $\frac{1}{3}$ that of a prism having the same base and altitude." (Euclid, XII. 7. 10.)

3. This is an important fact to remember, and may be thus practically illustrated: Take a cube or square prism of cork, clay, or other convenient substance. Draw a line joining the centres of two opposite edges of any side, and from this as the *ridge* cut away slantingly on each side a wedge-like piece down to the edge of the base. Each of these will evidently be $\frac{1}{3}$ the whole mass, and therefore *just half* will remain, presenting the appearance of a ROOF with 2 gables. Now bisect the ridge, and from the point of bisection, again cut slantingly down to the base's edge on both sides, removing the gables, and thus forming a *pyramid* whose base and height are the same as those of the prism out of which it has been formed. Lastly, bisect the pyramid by cutting it vertically down through two opposite angles, and the result is 2 triangular pieces, each of which will be found to be exactly similar and equal to the triangular piece formed *by joining the two gable ends* previously cut off; *i. e.* each half of the pyramid $= \frac{1}{3}$ whole roof, or $\frac{1}{3}$ of $\frac{1}{2}$ whole prism; $\therefore$ whole pyramid $= \frac{2}{3}$ of $\frac{1}{2}$ prism $= \frac{1}{3}$ *whole prism.* Q. E. I.

4. The cone is but a *pyramid on a circular base*, and accordingly its solidity is $\frac{1}{3}$ that of a cylinder having the same base and height. (*Vid.* Prism, Note 2.)

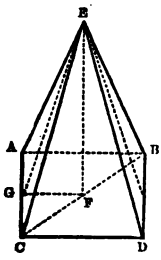
Find solidity of the following pyramids and cones: —

1. Square base. Side 27 yds. 2 ft.; height 96 yds.
Ans. 24494 yds. 6 ft.
2. Regular 3n. Side 5 ft. 4 in.; height 13 ft. 9 in.
Ans. 2 yds. 2 ft. 780 in.
3. Scalene △. Sides 76, 58, 64 yds.; height 125$\frac{1}{2}$ yds.
Ans. 75668$\frac{1}{8}$ yds.
4. Hexagon. Side, 36 yds.; height 219 yds. *Ans.* 3465296$\frac{5}{16}$
5. 8n. Side 36 yds.; height 177$\frac{3}{4}$ yds. *Ans.* 375041 yds.
6. 8n. Side 24 ft.; height 74·25 yds. *Ans.* 7635 yds.
7. 5n. Side 63 yds.; radius circumscribed ⊙ 54 yds.; height 237 yds.
Ans. 545712$\frac{1}{4}$ yds.
8. 5n. Side 37 yds., ⊥ from ∠ to opposite side 58 yds.; height 186 yds. *Ans.* 438123 yds.
9. Circle. Diameter 16 yds. 1 ft.; height 78·84 yds.
Ans. 49577·22 yds.
10. Circle. ○ce 16 ft. 6 in.; height 48·78 yds.
Ans. 39 yds. 3 ft. 677 in.

THE SQUARE PYRAMID.

Case I. This is the ordinary case, which has been already dealt with, of determining the pyramid's solidity from the *base-length and height*, which readily yield the desired result. However, if only one or neither of these dimensions can be ascertained directly, the requisite data must be arrived at circuitously, or, as it were, by stratagem; and the following cases exhibit several easy problems (all depending on Euclid, I. 47.), which it is hoped the student will by this time be easily able to solve for himself.

Fig. 46.



N. B. Since E F stands upright on the plane surface

A B C D and is ⊥ to it, it is also ⊥ to every line (as G F, B C) which meets it in that plane. (Euclid, XI.)

Case II. Given, height and diagonal of base: find solidity.

	EF.	*BC.*	*Ans. Solidity.*
1.	25 yds.	70 yds.	20416 yds. $17\frac{1}{2}$ ft.
2.	30 yds.	78 yds.	30420 yds.
3.	47 ft.	89 ft.	2298 yds. $1\frac{1}{2}$ ft.
4.	$7\frac{3}{4}$ ft.	$17\frac{1}{2}$ ft.	14 yds. 17 ft. 989 in.

Case III. Given, height and shortest distance from apex to base: find solidity.

	EF.	*EG.*	*Ans. Solidity.*
1.	13 in.	18 in.	1 ft. 958 in.
2.	30 ft.	50 ft.	2370 yds. 10 ft.
3.	40 yds.	75 yds.	214660 yds. $17\frac{3}{10}$ ft.
4.	15 p.	25 p.	1331000 yds.

Case IV. Given, height and angular edge: find solidity.

	EF.	*EC.*	*Ans. Solidity.*
1.	17 in.	29 in.	3 ft. 1072 in.
2.	25 ft.	50 ft.	1157 yds. 11 ft.
3.	19 yds.	27 yds.	4661 yds. 9 ft.
4.	13 p.	18 p.	223497 yds. 2 ft. 432 in.

Case V. Given, length of base and shortest distance from it to apex: find solidity.

	AC.	*EG.*	*Ans. Solidity.*
1.	28 ft.	56 ft.	674 yds. 16 ft.
2.	75 yds.	175 yds.	320503 yds.
3.	4 ft. 10 in.	13 ft. 2 in.	3 yds. 19 ft. 1361 in.
4.	87 yds.	178 yds.	435470 yds.

Case VI. Given, base-length and angular edge: find solidity.

	AC.	*EC.*	*Ans. Solidity.*
1.	40 yds.	70 yds.	$34149\frac{4}{5}$ yds.
2.	$17\frac{1}{2}$ ft.	$33\frac{3}{4}$ ft.	118 yds. 19·3146 ft.
3.	16 p.	35 p.	2826·325 p.
4.	70 yds.	156 yds.	241633·7 yds.

Case VII. Given, E G and E C: find solidity.

	EG.	*EC.*	*Ans. Solidity.*
1.	75 yds.	95 yds.	214817 yds. 10 ft.
2.	4 ft. 9 in.	6 ft. 5 in.	1 yd. 22 ft. 567 in.
3.	25 ft.	35 ft.	148 yds. $3\frac{7}{10}$ ft.
4.	36 yds.	48 yds.	22807 yds. $18\frac{1}{3}$ ft.

Case VIII. Given, diagonal of base and angular edge: find solidity.

	BC.	*EC.*	*Ans. Solidity.*
1.	78 yds.	78 yds.	68494 yds. $18\frac{1}{2}$ ft.
2.	3 ft. 10 in.	7 ft. 10 in.	18 ft. 1039 in.
3.	68 ft.	97 ft.	2593 yds. 1 ft.
4.	6 ft. 2 in.	15 ft. 7 in.	3 yds. 15 ft. 1404 in.
5.	40 yds.	40 yds.	9237·6 yds.

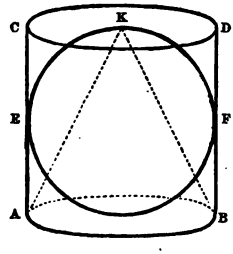
Case IX. Given, B C and E G: find solidity.

	BC.	*EG.*	*Ans. Solidity.*
1.	60 yds.	30 yds.	12727 yds. $21\frac{1}{3}$ ft.
2.	84 yds.	48 yds.	44335 yds. $5\frac{2}{3}$ ft.
3.	$28\frac{3}{4}$ ft.	$17\frac{1}{2}$ ft.	72 yds. $18\frac{1}{4}$ ft.
4.	150 yds.	75 yds.	198873$\frac{3}{4}$ yds.

THE SPHERE.

1. The Sphere or ball is found, by calculation and experiment, to be $\frac{2}{3}$ of its circumscribing cylinder, *i. e.* of the cylinder whose diameter and height are the same; just as the *cone* is found to be $\frac{1}{3}$ ditto.

Fig. 47.



This may be illustrated in the following way: let A B C D be a 3-pint cylinder full of water. Immerse carefully the cone K A B, and 1 pint will run over. Then remove the cone and substitute the sphere K E F, and another pint will

be found to escape,—proving that the ball occupies the place of 2 pints out of three, *i. e.* $\frac{2}{3}$ whole cylinder.

2. There is no difficulty, therefore, in calculating its solidity. For let diameter $= x$.

Then, since solidity of cylinder = area of base × height, and since in this case base and height are equal,

$$\therefore \text{Solidity of cylinder} = \frac{3\frac{1}{7}}{4}x^2 \times x = \frac{3\frac{1}{7}}{4}x^3.$$

$$\therefore \text{Solidity of sphere} = \tfrac{2}{3} \text{ of } \frac{3\frac{1}{7}}{4}x^3 = \frac{3\frac{1}{7}}{6}x^3 = \frac{11}{21}x^3.$$

3. When the question concerns a *hollow sphere*, the solidity of its crust or shell is readily determined by finding, first, the content of the *whole sphere* as a *solid*, and then of the *interior* as ditto, and subtracting the latter from the former. This difference is, of course, the solidity of the shell.

4. The bore of a cylinder, or calibre of a gun, requisite to carry a ball of any given weight, is ascertained by finding the number of cubic inches in the ball, and then obtaining its diameter (Note 2.).

5. When the gun's calibre is known, the weight of ball carried is found by reversing this process.

6. By reference to the Solid, Notes 10—12., and Circle, Note 18., the student will be reminded of the fact, that "the solidity of the sphere increases or diminishes as the *cube* of its diameter;" and will also see what change must be made in diameter or circumference of a ball in order to effect any requisite increase or diminution in its solidity, *i. e.* its size, weight, &c. (Euclid, XII. 18.)

Case I. Given, diameter: find solidity (*Vid.* notes).

(1). 3 ft. 6 in.	(2). $10\frac{1}{2}$ yds.	(3). 5 yds. 2 ft. 6 in.
(4). 4 ft. 8 in.	(5). 126 yds.	(6). 1 yd. 2 ft. 6 in.

Answers.

22 ft. 792 in.	606 yds. 10 ft. 216 in.	103 yds. 26 ft. 504 in.
1 yd. 26 ft. $405\frac{1}{2}$ in.	1047816 yds.	22 yds. 16 ft. 72 in.

Case II. Given, circumference: find solidity.

(1.) 5 ft. 6 in. (2.) 12 yds. 2 ft. 6 in. (3.) $52\frac{1}{2}$ yds.
(4.) 24640 miles. (5.) 87 yds. 1 ft. 6 in. (6.) $4\frac{1}{2}$ yds.

Answers.

2 ft. 1395 in. 35 yds. 17 ft. 1557 in. 2313 yds. 0 ft. $712\frac{35}{121}$ in.
$252418730666\frac{2}{3}$ miles. 3101 yds. 13 ft. 405 in. 1 yd. 6 ft. nearly.

Case III. Given, solidity: find diameter. (Note 4.)

N.B. Let S represent solidity, and D diameter,

$$\text{then } S=\frac{11}{21}D^3 \quad \therefore D=\sqrt[3]{\frac{21S}{11}}.$$

(1.) 4851 in. (2.) 1047816 in. (3.) 38808 yds.
(4.) 1054152 yds. (5.) 1663893 ft. (6.) $606\frac{3}{4}$ in.

Answers.

1 ft. 9 in. $3\frac{1}{2}$ yds. 42 yds.
66 yds. 147 ft. $10\frac{1}{2}$ in.

Case IV.

1. Find weight and circumference of two iron balls, whese diameters are respectively twice and thrice that of a 9 lb. shot. (Note 6.)

Ans. $\begin{cases} 87\frac{6}{10} \text{ and } 294\frac{7}{10} \text{ lbs.} \\ 26\frac{2}{3} \text{ and } 39\frac{2}{3} \text{ inches.} \end{cases}$

2. How much iron is there in a-10 inch shell made of iron $1\frac{1}{2}$ inch thick? *Ans.* $96\frac{3}{4}$ lbs.

3. How many 8-inch shells, iron $\frac{1}{2}$ inch thick, can be made out of 20 tons of metal? *Ans.* 1802.

4. A 13-inch shell, $1\frac{1}{4}$ inch iron, is filled with lead. How much heavier will it be than if it were solid iron? *Ans.* $79\frac{1}{8}$ lbs.

5. Given, that the bore (or diameter) of a 9-pounder is 4·2 inches, find bore of two other guns which throw balls of twice and thrice the weight respectively. *Ans.* 5·29; 6·0564 inches.

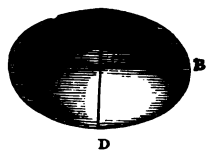
6. Given, that the calibre (diameter) of a 24-pounder is 5·82 inches, find calibre of two other guns which carry 12 and 6 lb. shot respectively. *Ans.* 4·62 and 3·66 inches.

7. The calibre of a gun being 2 inches, find weights of the largest iron and leaden ball which it will carry. *Ans.* $18\frac{1}{7}$ and $27\frac{3}{28}$ os.

THE SPHEROID.

1. The spheroid, when shaped as an *egg*, is produced by the revolution of the ellipse round its *major axis* A B, which remains fixed or stationary; and is called "*prolate*" or oblong.

Fig. 48.



2. When shaped as an *orange*, it is produced by the ellipse revolving round C D, its *minor axis*, which remains fixed; and is called "*oblate.*" This is the shape of the earth and other planets.

3. The spheroid, like the sphere, is $\frac{2}{3}$ of its circumscribing cylinder.

4. The spheroid, being closely allied to the sphere, follows the same law of calculation; so that, as solidity sphere = $\frac{3\frac{1}{7}}{6}$ (diameter^2 × diameter); so solidity spheroid = $\frac{3\frac{1}{7}}{6}$ (revolving axis^2 × fixed axis).

Case I. *Prolate.* Given, major and minor axes: find solidity. (*Vide* Notes.)

	Major Axis.	*Minor Axis.*	*Ans. Solidity.*
1.	48 yds.	35 yds.	30800 yds.
2.	7 ft. 7 in.	3 ft. 3 in.	1 yd. 14 ft. 1653 in.
3.	$19\frac{1}{2}$ ft.	$15\frac{3}{4}$ ft.	93 yds. $22\frac{1}{2}$ ft.
4.	$25\frac{5}{8}$ yds.	$13\frac{1}{2}$ yds.	2450 yds. 6 ft. 1296 in.

Case II. *Oblate.* Given, major and minor axes: find solidity.

	Major Axis.	*Minor Axis.*	*Ans. Solidity.*
1.	5 ft. 3 in.	2 ft. 4 in.	1 yd. 6 ft. 1188 in.
2.	$29\frac{3}{4}$ yds.	$19\frac{1}{2}$ yds.	9040 yds. $7\frac{3}{5}$ ft.
3.	$17\frac{5}{8}$ ft.	$8\frac{3}{4}$ ft.	1423 yds. $20\frac{4}{5}$ ft.
4.	4 ft. $4\frac{1}{2}$ in.	1 ft. $5\frac{1}{2}$ in.	14 ft. $1073\frac{5}{8}$ in.

Case III. Given, solidity and fixed axis: find revolving axis. (*Vide* Notes.)

[*N. B.* Determine the character of each spheroid, *i. e.* whether prolate or oblate.]

	Solidity.	*Fixed Axis.*	*Ans. Revolver.*
1.	2533·8 ft.	6 yds. 1 ft. 6 in.	5 yds. 0 ft. 9 in.
2.	58212 in.	28 in.	63 in.
3.	1140 yds. 20 ft.	144 ft.	105 ft.
4.	244087·6 ft.	89¼ ft.	19½ yds.

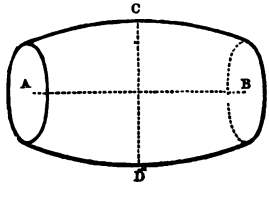
Case IV. Given, solidity and revolving axis: find fixed axis.

	Solidity.	*Revolving Axis.*	*Ans. Fixture.*
1.	1 yd. 14 ft. 1653 in.	39 in.	91 in.
2.	38441·8 ft.	17·625 ft.	8·75 ft.
3.	25265·625 in.	52½ in.	17½ in.
4.	2450 yds. 6 ft. 1296 in.	13 yds. 1 ft. 6 in.	25 yds. 2 ft.

THE BARREL OR CASK.

1. The Barrel may be considered as the "middle frustum" of a spheroid, *i. e.* as a spheroid with its ends cut off *vertically* by planes passing through it *perpendicular to the major axis.*

Fig. 49.



2. Content of the cask = *mean area of vertical section* × *length*, or distance between the two heads of the cask.

3. "Mean area" is found in practice to be determined with sufficient accuracy by taking the *average of* 3 *vertical sections*, viz., one at the head or end, and two at the bung or centre.

e. g. Diameter at head = 14 inches, and circumference at bung = 48 inches, while length = 14½ inches, (thickness of wood having been allowed for).

Now "mean area"
= ⅓ (area at head + 2 area at bung).
= ⅓ (154 + 366·5454 inches) by question.
= 173·5 inches.

∴ Content of cask = 173·5 x 14·5 = 2516 cubic inches.

And since one gallon contains 277 cubic inches,

$$\therefore \text{Gallons in cask} = \frac{2516}{277} = 9 \text{ nearly.}$$

Given, circumference at head and bung, and length or depth: find contents. (*Vide* Notes.)

	Head ○ce.	*Bung* ○ce.	*Length.*	*Ans. Contents.*
1.	50⅞ in.	59⅛ in.	21 in.	18 gals.
2.	7 ft. 4 in.	8 ft. 4 in.	27 in.	72 gals.
3.	5 ft. 6 in.	6 ft. 2 in.	24½ in.	36 gals.
4.	11 ft. 5½ in.	12 ft. 10 in.	4 ft. 2 in.	316 gals.
5.	16½ ft.	18½ ft.	5 ft. 3 in.	828 gals.
6.	14 ft. 8 in.	16 ft. 8 in.	4 ft. 6 in.	572 gals.
7.	22 ft.	27½ ft.	8 ft. 6 in.	2779 gals.

[*Obs.*—The above are *internal* dimensions, guessed at from *external measurement*. The results therefore are not quite exact.]

THE SOLID RING.

1. This is but the frame or zone of "Plane Mensuration," with the addition of *depth* or thickness; and its content or solidity must accordingly be estimated on the same principles.

2. When the ring is a *flat* one, *i. e.* formed of a rectangular bar or rod bent round, its solidity = area of upper sur-

ace × thickness, (the ring being supposed to lie in a horizontal position).

3. When the ring is cylindrical, *i. e.* formed of a cylinder bent round, its solidity = area of a vertical section × mean circumference. The flat ring can, of course, be calculated in the same way.

4. A "vertical section" of any solid is formed by a plane cutting through it in a direction perpendicular or vertical to the horizon; and a "horizontal section" is parallel to the horizon.

5. For mean perimeter and diameter, *vid.* "Frame or Ring" in Plane Mensuration, *suprà*.

1. The outer and inner diameters of a ring are $27\frac{1}{2}$ and $19\frac{1}{2}$ inches, and area of vertical section is 1 ft. 73 in.: find solidity.

Ans. 9 ft. 475 in.

2. The outer and inner diameters of a ring are $3\frac{1}{2}$ and $2\frac{1}{4}$ yds., and area vertical section is 2 yds. 7 ft. 89 in.: find solidity.

Ans. 8 yds. 4 ft. $1651\frac{1}{2}$ in.

3. A rectangular bar of metal is bent into a wheel. Its outer and inner diameters are 44 and 26 inches, and thickness 6 inches. Find solidity. *Ans.* 3 ft. 756 in.

4. Breadth of flat ring is $4\frac{1}{2}$ ft., and thickness 1 yd., inner diameter being 13 ft.: find content. *Ans.* $27\frac{1}{2}$ yds.

5. Outer and inner diameters of a cylindrical ring are 9 and 5 inches: and solidity. *Ans.* $69\frac{1}{7}$ in.

6. Outer and inner diameters of a cylindrical ring are 16 and 12 ft.: and solidity. *Ans.* $138\frac{1}{4}$ ft.

7. The circumference of the cylinder forming a ring is 2 ft. 9 in., and *inner* diameter of ring is 2 ft. 4 in.: find solidity. *Ans.* 6 ft. $113\frac{5}{8}$ in.

8. The circumference of the cylinder is 1 ft. 10 in., and *outer* diameter ft. 10 in.: find solidity. *Ans.* 1 ft. $1539\frac{1}{2}$ in.

9. Diameter or thickness of a solid cylindrical ring of elliptic shape 1 ft. 9 in.; and its major and minor axes measured *internally* are 6 ft. in. and 4 ft. 8 in.: find solidity. *Ans.* 2 yds. 1 ft. $247\frac{1}{2}$ in.

THE PIPE OR HOLLOW CYLINDER.

1. This is one of the most common and important shapes in which the solid ring meets the eye,—length and calibre (*i. e.* diameter of bore) being its most noticeable features (Euc. XII.)

Fig. 50.

2. The weight or quantity of metal, &c. in a yard, &c. of piping, and consequently its *value*, is determined by finding the area of its vertical section, and multiplying this by the particular length in question. This gives *cubic* inches, feet, yards, &c.: and the weight of a cub. inch being known (as *iron* $4\frac{1}{3}$ oz., and *lead* $6\frac{3}{7}$ oz.), the weight, &c. of the whole piping is readily ascertained.

3. When it is required to discover what length, calibre, or thickness of material in piping, can be manufactured out of a given mass of clay, metal, &c., the desired result is attained by *dividing the cubic content of the mass by the product of the two given dimensions.* (Parallelopiped, *Case* II.)

4. The student must particularly bear in mind the important fact that the *volume* of the cylinder, *i. e.* its power of containing or discharging fluid, &c. varies as the *square* of its diameter. (*Vid.* Circle, Notes 17 and 18.)

5. When fluid is passing through any cylinder, its *velocity* supplies a third dimension (*viz.* length) to the 2 which form the area of the cylinder's mouth. So that when the quantity discharged in an hour or other period is known, *the rate per hour* at which it has flowed through is known also, by dividing the space which it occupies by the area of a vertical section of the cylinder.

6. And as *volume* or content of cylinder varies as the square of its diameter, so does the velocity of any fluid passing through it, but in *inverse proportion.* For if the fluid be impelled by pressure, a certain quantity *must* find a passage whether the pipe's bore be large or small; and the

same volume cannot flow through a narrow and a wide orifice in the same time, except by increasing its speed through the former, or slackening it through the latter, as the case may require. And this increase or diminution, therefore, will evidently be in *inverse proportion* to the diameter or area passed through—the impelling force remaining the same.

7. Thus, if a cylinder be 12 inches diameter at one end and 6 at the other, and water flowing 10 miles an hour enter the large end, then, since area of small end is only ¼ area of the large, and since the same quantity of water that goes in must come out *in the same time* under the constant influence of the pressure from behind,—it follows that the fluid must escape with 4 times the speed at which it entered, *i. e.* 40 miles an hour. It is on this principle that fire-engines, and all the various modifications of the squirt, are constructed, and on which all their value depends.

8. If the same water had entered the *small* end and escaped at the *large*, the direct reverse of all this would of course have happened, and the velocity at the outfall would have been just ¼ of 10, *i. e.*, 2½ miles per hour. For fluids *press equally in all directions*, and since, in its passage from the small orifice to the large one, the water finds continually more room for expansion, not only by direct advance as before, but above, below, and on each side, the force with which it entered is spent not only in rushing forward, but also in diffusing itself over this new area, so that its velocity is continually diminishing in proportion to the size of this area, *i. e.* in proportion to the square of the pipe's diameter.

9. Or thus: the stream having at its outfall 3 times *more* space to occupy than was accessible at its entrance, 3 parts out of 4 of its initial force will be expended in filling up these 3 new portions of space, leaving but ¼ for the *forward* progress of the fluid; while in the reverse case (Note 7.), water sufficient to fill 4 areas being compressed into *one*, the force acting upon the fluid in each (*i. e.* 4 forces in all) is concentrated, and in consequence the new velocity pro-

duced by this combination becomes quadruple what it was at the commencement.

10. The operation of this principle may be witnessed in the flow of almost any river, which is alternately gentle and rapid according as the channel is deep or shallow, expansive or confined.

1. The outer and inner diameters of an iron pipe are 12¼ and 10 inches: find its weight per yd. *Ans.* 2 cwt. 3 qrs. 9 lbs. nearly

2. Iron piping is an inch thick and 1 yd. in diameter internally find cost of 5 miles such piping, at £7 per ton, giving the more exact weight of 4·526 oz. to the cubic inch. *Ans.* £32565 7*s.* 6½*d*

3. What length of 8-inch pipe (extreme diameter), made of ¼ inch iron, can be manufactured out of 100 tons of metal? *Ans.* 1877¼ yds

4. Pipe *A*, of 2-inch bore, discharges 20 gals. per minute. How much will flow in the same time through *B* and *C*, whose diameters are respectively twice and thrice that of *A*? *Ans.* 80 and 180 gals

5. Hourly discharge of a 3-inch pipe is 2400 gals.: find quantities discharged in 20 minutes by two pipes of 4 and 2 inches diameter respectively. *Ans.* 1422⅔; 355⅔ gals

6. A 5-inch pipe discharges 70 gals. per minute. What must be the bore of two others which discharge twice and thrice the quantity in same time? *Ans.* 7·071 and 8·66 inches

7. 6000 and 4000 gals. flow in an hour through a 6- and 4-inch pipe respectively: find velocity of each.

Ans. 1632½ yds. and 2448¾ yds. per hr.

8. Diameter of pipe *A* is 6-inch at one end and 3 at the other; of pipe *B* is 6 do. and 2 do. Water flowing 6 miles an hour enters each at the large end. With what velocity will it escape from each?

Ans. 24 and 54 miles per hr.

9. If the water, however, were to enter at the *small* end and escape at the *large*, what would be its velocity at the outfall?

Ans. 1½ and ⅔ mile per hr.

THE FRUSTUM OF A PYRAMID, CONE, OR WEDGE.

1. The "Frustum" is a portion of a solid.

2. The frustum of a pyramid, cone, or wedge, is that portion which remains after the top or thin end has been cut *off by a plane* running parallel to the base.

3. An *obelisk* and *mast* are frustums of a pyramid and cone respectively.

4. The frustum of a wedge is called a prismoid, the *base*

Fig. 51.

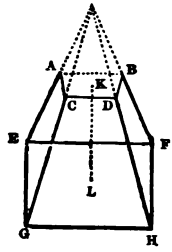


Fig. 52.

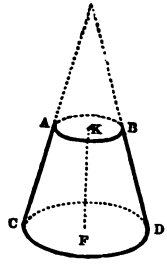
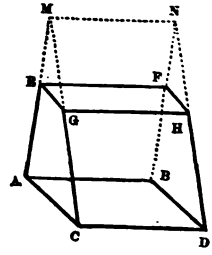


Fig. 53.



being its rectangular back. Such is a railway cutting or embankment, canal, trough, &c.

5. To determine the solidity of any of these, it is in general sufficient to obtain the mean area of the 2 parallel ends, and multiply this by the ⊥ distance between them.

6. It is found, however, by experiment, that where great accuracy is required, the "mean area" is obtained more exactly thus: —

α. Add together area of base, area of top, and 4 times area of section parallel to and midway between the two ends (*i. e.* 6 areas in all), and take $\frac{1}{6}$ the sum.

β. To areas of 2 parallel ends add the *mean proportional* between them (*i. e.* 3 areas in all), and take $\frac{1}{3}$ the sum. Either method gives same result.

e. g. A square pyramidical frustum has its sides at base and top = 7 and 5 yards, and height 36 yards: find solidity.

First method. Mean area $= \frac{49+25}{2} = 37$ sq. yds.

$\therefore$ Solidity $= 37 \times 36 = 1332$ cubic yds.

Second method. (*a*) Mean area $= \dfrac{49 + 25 + 4\left(\frac{7+5}{2} \times \frac{7+5}{2}\right)}{6}$

$= \dfrac{218}{6} = 36\frac{1}{3}$ square yds.

$\therefore$ Solidity $= 36\frac{1}{3} \times 36 = 1308$ cubic yds.

Ditto. (β.) Mean area $= \dfrac{49 + 25 + \sqrt{49 \times 25}}{3} = \dfrac{109}{3} = 36\frac{1}{3}$.

$\therefore$ Solidity = 1308 yards, as before.

7. It will be noticed in the foregoing example, that the result by the first method differs only 24 yards from that by the second; and that the error is one, not of *defect*, but *excess*, which is always the safe side to take in making estimates.

In the following examples, the more exact method is adopted:—

1. In frustum of square pyramid, sides of base and top are 21 and 15 yds., while height is 84 yds.: find solidity. *Ans.* 27468 yds.
2. Do. do. 36 and 18 yds., height 120 yds.: find do. *Ans.* 90720 yds.
3. Do. do. 45 and 25 yds., height 96 yds.: find do. *Ans.* 120800 yds.
4. In frustum of a cone, circumferences of base and top are 5 ft. 6 in. and 1 ft. 10 in., height 18 ft.: find solidity. *Ans.* 20 ft. 1476 in.
5. Do. do. 4 ft. 7 in. and 2 ft. 9 in., height 8 yds.: find do. *Ans.* 26 ft. 348 in.
6. Do. do. 86 and 68 yds., height 57 yds.: find do. *Ans.* 27005 yds.
7. Prismoid. Base (24 × 18) yds., top (24 × 12) yds., height 63 yds.: find do. *Ans.* 22527½ yds.
8. Do. do. (36 × 25) and (28 × 19) ft., height 27 yds.: find do. *Ans.* 57346½ ft.
9. Do. do. (17 × 15) and (19 × 14) inches, height 23 inches: find do. *Ans.* 3 ft. 807 in.

THE IRREGULAR PRISM.

To find the solidity of an irregular prism, as block of stone, trunk of tree, &c.

Take circumference, breadth and depth, or other measurements, not only at the 2 ends, but, at various points *equally distant* from each other. Find the average of all these, and from this calculate the average or *mean area* of a vertical section, which multiply by the *length*, for solidity.

e. g. Trunk of tree is 64 ft. long; and 5 equidistant circumferences, (including the 2 ends) are $14\frac{1}{2}$, $13\frac{1}{4}$, $11\frac{1}{4}$, $8\frac{3}{4}$, $7\frac{1}{4}$ ft.

$\therefore$ Mean circumference = $\frac{1}{5}$ of 55 ft. = 11 ft.
$\therefore$ Mean area = $9\frac{5}{8}$ sq. ft.
$\therefore$ Solidity = $9\frac{5}{8} \times 64 = 616$ cub. ft.

1. A pole is $29\frac{1}{2}$ ft. long, and 5 equidistant circumferences are $29\frac{3}{4}$, $26\frac{1}{2}$, $21\frac{1}{4}$, $16\frac{3}{4}$, $15\frac{3}{4}$ inches. Find solidity. *Ans.* 7 ft. 1533 in.

2. A block of stone is $17\frac{3}{4}$ ft. long, and 4 equidistant dimensions are $(3\frac{1}{2} \times 2\frac{3}{4})$ ft., $(3\frac{1}{4} \times 2\frac{1}{2})$ ft., $(2\frac{3}{4} \times 2\frac{1}{4})$ ft., $(1\frac{1}{2} \times 1\frac{1}{4})$ ft. Find solidity. *Ans.* 4 yds. $6\frac{9}{16}$ ft.

3. A tree is 78 ft. long, and 6 circumferences are $17\frac{1}{2}$, $14\frac{1}{4}$, $12\frac{1}{2}$, $10\frac{1}{2}$, $9\frac{3}{4}$, $7\frac{1}{2}$ ft. Find value at £2 9*s.* 6*d.* a cub. yd. *Ans.* £81 18*s.*

4. A mass of Caen stone is $13\frac{1}{2}$ ft. long; and 4 dimensions are (29×26), (26×25), (23×19), (18×15) in. Find value at 3*s.* a cub. ft. *Ans.* £7 8*s.* $5\frac{5}{33}$*d.*

PILING OF SHOT AND SHELL.

Fig. 54.

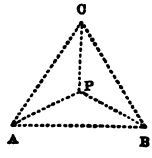


Fig. 55.

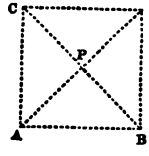
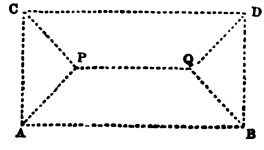


Fig. 56.



1. For economy of space and general convenience, shot and shell are usually stored in *piles;* the pile consisting of

horizontal courses, and the base forming either an equilateral triangle, square, or oblong.

2. The triangle and square are regular pyramids terminating in P, a single ball; while the oblong has P Q, a row of balls at top, which is always *one more than the difference between the length and breadth of base.*

3. To determine the number of balls in a pile. Find the number in the 3 parallel edges, and also in the triangular face. Multiply these two results together, and $\frac{1}{3}$ product will be the number required.

[*N.B.* pyramid = $\frac{1}{3}$ prism.]

4. Observe the following particulars.

α. In the triangle, the *parallel edges* are A B + P + C (P and C being each 1 ball).

β. In the square, ditto, are A B + C D + P.

γ. In the oblong, ditto, are A B + C D + P Q.

δ. No. of balls in triangular face = $\frac{1}{2}$ base × (base + 1).

ε. No. of courses = No. of balls in the breadth of base.

5. When the pile is a frustum or incomplete pyramid, the number of balls in it may be found by calculating the pile as complete, and then calculating in the same way the upper or absent portion as a complete pyramid too. The difference between these two is the content of the frustum.

Ex. Find balls in a triangular pile, base 18.

Here, sum of parallel edges, = 18 + 1 + 1 = 20.

Balls in triangular face = $\frac{18}{2}$ × (18 + 1) = 171.

∴ Balls in pile = $\frac{1}{3}$(20 × 171) = 1140.

EXAMPLES.

1. Find number of balls in each of 3 triangular piles, whose bases number respectively 14, 17, 21 balls. *Ans.* 560, 969, 1771.

2. Find number in each of 3 square piles, whose bases are 10, 13, 19 balls. (Note 4. *β.*) *Ans.* 385, 819, 2470.

3. Find number in each of 3 oblong piles, whose dimensions at base are (9 × 5), (13 × 10), (20 × 15) balls. (Note 2 and 4. *γ.*)

Ans. 115, 550, 1840.

4. How many more balls are there in a *square* than a *triangular* pile, base of each being 15 balls? *Ans.* 560.

5. Find total number of balls in 3 piles, viz. *A*, triangular, base 13; *B*, square, base 15; *C*, oblong, bases (13 × 15) balls. *Ans.* 2696.

6. Find number in the frustum of a triangular pile, whose bases at bottom and top are 16 and 7 balls. *Ans.* 732.

7. Find number in frustum of square pile, whose bases at bottom and top are 16 and 7 balls. *Ans.* 1356.

8. Find number in frustum of oblong pile, whose bases at bottom and top are (18 × 13) and (13 × 8 balls). *Ans.* 890.

PART III.

THE SURFACE OF SOLIDS.

THE PARALLELOPIPED.

Given, the 3 dimensions; find area of its 6 sides.

	Length.	*Breadth.*	*Depth.*	*Ans. Area.*
1.	9 in.	4 in.	3 in.	1 ft. 6 in.
2.	17 ft.	15 ft.	13 ft.	149 yds. 1 ft.
3.	13½ in.	11½ in.	9½ in.	5 ft. 65½ in.
4.	17 yds. 2 ft.	15 yds. 1 ft.	13 yds. 2 ft.	1443 yds. 7 ft.
5.	19¾ ft.	16½ ft.	13¼ ft.	179 yds. 1 ft. 54 in.
6.	26½ yds.	21¾ yds.	17½ yds.	2841 yds. 4 ft. 72 in.
7.	5½ ft.	4 ft. 8 in.	2 ft. 9 in.	11 yds. 8 ft. 36 in.
8.	4 ft. 3 in.	3 ft. 6 in.	2 ft. 7 in.	7 yds. 6 ft. 114 in.

[For a description of this and other figures, *vid.* "Solids."]

THE PRISM

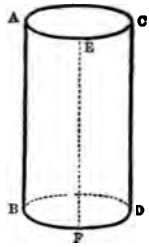
Given, fig. of base, length of side, and height; find area of all the sides.

	Fig. base.	*Side.*	*Height.*	*Ans. Area.*
1.	3in.	7 in.	13 in.	1 ft. 129 in.
2.	4in.	13½ in.	29½ in.	1 yd. 1 ft. 139½ in.
3.	5in.	23½ in.	7½ ft.	9 yds. 1 ft. 63 in.
4.	6in.	12½ ft.	32¾ yds.	808 yds. 4 ft. 72 in.
5.	7in.	11¾ in.	3½ ft.	2 yds. 6 ft. 122 in.
6.	8in.	5¾ p.	15¾ p.	4 a. 2 r. 3 p. 21 yds.
7.	9in.	5 c. 75 lks.	11 c. 25 lks.	58 a. 2 c. 1875 lks.
8.	10in.	17½ in.	13½ ft.	22 yds. 1 ft. 87¾ in.

[As with the Cube, the area of a Prism = sum of areas of its several sides, which are all □s.]

THE CYLINDER.

Fig. 57.



If the hollow cylinder A B C D be cut down vertically at any one side, as in E F, and opened out flat, the material will form a rectangular ▱. The area, therefore, of a cylindrical surface = perimeter × height.

1. Three hollow cylinders, each 5 ft. 9 in. high, have perimeters 4 ft. 8 in., 3 ft. 7 in., and 2 ft. 6 in. Find convex (*i.e.* outside or external) area of each.

Ans. 26 ft. 120 in.; 20 ft. 87 in.; 14 ft. 54 in.

2. Three solid cylinders, each 6 ft. 10 in. high, have perimeters 5 ft. 9 in., 4 ft. 10 in., 3 ft. 11 in.: find total cylindrical area, and also total area of their flat ends. *Ans.* 11 yds. 0 ft. 12 in.; 5 ft. 102 in.

3. Three solid cylinders, each 4 ft. 9 in. high, have diameters 1 ft. 9 in., 17½ in., 16 in.: find total area of each.

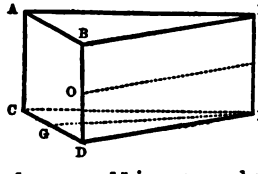
Ans. { 1. 3 yds. 3 ft. 135 in.
2. 2 yds. 7 ft. 16½ in.
3. 2 yds. 4 ft. 100½ in.

4. Five solid cylinders, each 7¾ ft. high, have base areas 2464 in., 1386 in., 3118½ in., 4658½ in., 6237 in.: find cylindrical area of each.

Ans. { 12 yds. 5 ft. 96 in.; 9 yds. 3 ft. 36 in.; 14 yds. 1 ft. 126 in.;
17 yds. 3 ft. 42 in.; 20 yds. 0 ft. 120 in.

THE WEDGE.

Fig. 58.



[*N. B.* A B C D is a rectangle; A B C is an isosc. △; and B D E F is a rectangle or trapezoid, according as E F is equal or unequal to B D.]

Find total surface in the following examples.

1. A B = 15 in.; A C and E F each 11 in.; F G, 23 in.

Ans. 7 ft. 34½ in.

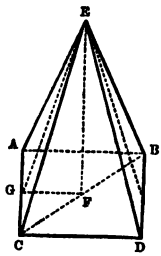
2. A B = 19 in.; A C and E F 13 in.; F G 27 in.

Ans. 10 ft. 64 in.

3. AB=27 yds.; BD and EF 38 yds.; BE 56 yds. *Ans.* 6749⅓ yds.
4. AB=38 ft.; BD and EF 29 ft.; AE 47 ft.
Ans. 606 yds. 7 ft. 79⅛ in.
5. CD=4 ft. 10 in.; AC 5 ft. 6 in.; EF 3 ft. 9 in.; OP 7 ft. 4 in. and FG 7 ft. *Ans.* 14 yds. 2 ft. 36 in.
6. CD= 17 in.; BD 13 in.; EF 19 in.; OP 25 in.; FG 2 ft.
Ans. 1 yd. 0 ft. 133 in.
7. CD=28 yds.; CA 37 yds.; EF 26 yds.; OP 57yds.; FG 55 yds.
Ans. 6167 yds.
8. CD=34 ft.; CA 29 ft.; EF 38 ft.; OP 76 ft.; FG 72 ft.
Ans. 947 yds. 3 ft.

THE PYRAMID.

Fig. 59.



[*N.B.* Since all the faces of a pyramid are △s, whose area depends upon *base* and *height*, *i. e.* AC and EG, the student must set himself to discover these from the data in each case given.

Case I. Given fig. of base, with AC and EG; find surface.

Fig. Base.	*Side.*	*Slope.*	*Ans. Area.*
1. 4n.	8 in.	1½ ft.	2 ft.
2. 3n.	1 ft. 6 in.	3½ ft.	7 ft. 126 in.
3. 5n.	3⅜ ft.	14¾ ft.	15 yds. 0 ft. 30 in.
4. 6n.	3 ft. 6 in.	13½ ft.	15 yds. 5 ft.
5. 7n.	7 ft. 8 in.	23 ft. 9 in.	70 yds. 7 ft. 42 in.
6. 8n.	8 yds. 2 ft.	39 yds. 1 ft.	1363 yds. 5 ft.
7. 9n.	37 yds. 1 ft.	175 yds. 2 ft.	29512 yds.
8. 12n.	7½ p.	37½ p.	10 a. 2 r. 7½ p.

Case II. Given, side and angular slope or edge, *i. e.* AC and EC: find surface. (*Vid. Case* I.)

Fig. Base.	*Side.*	*Edge.*	*Ans. Area.*
1. 5n.	4 ft. 2 in.	19 ft. 7 in.	22 yds. 4 ft. 120¼ in.
2. 6n.	7½ ft.	57½ ft.	143 yds. 3 ft. 143¾ in.
3. 7n.	9¼ ft.	65 ft. 9 in.	243 yds. 6 ft. 55 in.
4. 9n.	17½ yds.	184¼ yds.	1493⅛ yds.
5. 11n.	8 ft. 4 in.	58¾ ft.	298 yds. 3 ft. 133 in.
6. 15n.	5½ p.	37½ p.	9 a. 2 r. 22 p. 21 yds.

Case III. *Square* pyramid. Given, A C and E F: find surface.

	AC.	*EF.*	*Ans. Area.*
1.	76 yds.	132 yds.	20878 yds. 6 ft. 69$\frac{1}{2}$ in.
2.	4 ft. 10 in.	9 ft. 6 in.	10 yds. 4 ft. 109$\frac{1}{12}$ in.
3.	24 p.	28 p.	1462 p. 6 yds. 7 ft.
4.	5 chns. 50 lks.	140 yds.	36908 yds.

Case IV. *Square.* Given, B C and E F: find ditto.

	BC.	*EF.*	*Ans. Area.*
1.	64 yds.	64 yds.	5143 yds. 6$\frac{1}{7}$ ft.
2.	5 ft. 8 in.	7 ft. 4 in.	6 yds. 6 ft. 132$\frac{8}{9}$ in.
3.	76 ft.	84 ft.	1053 yds. 1$\frac{7}{8}$ ft.
4.	7 c. 50 l.	18 p.	20293 yds.

Case V. *Square.* Given B C and E C: find ditto.

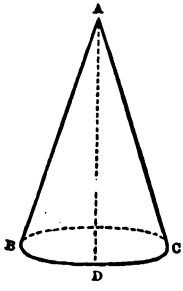
	BC.	*EC.*	*Ans. Area.*
1.	40 yds.	40 yds.	2116·577 yds.
2.	46 ft.	64 ft.	447 yds. 3 ft. 77$\frac{2}{3}$ in.
3.	5 ft. 10 in.	7 ft. 4 in.	6 yds. 3 ft. 152 in.
4.	6 c. 50 lks.	28 p.	29417$\frac{1}{2}$ yds.

Case VI. *Square.* Given, base area, and height: find ditto.

	Base Area.	*Height.*	*Ans. Surface.*
1.	9216 yds.	96 yds.	20607$\frac{1}{2}$ yds.
2.	1 r. 1 p. 28 yds. 4 ft.	75 yds.	5491 yds. 3$\frac{1}{2}$ ft.
3.	702 yds. 2 ft. 36 in.	26$\frac{1}{2}$ yds.	1570 yds. 2 ft. 72$\frac{1}{2}$ in.
4.	5 a. 1 r. 1 p.	39 p.	15 a. 0 r. 13$\frac{1}{4}$ p.

THE CONE.

Fig. 60.



If the hollow cone A B C be cut vertically down one side, as in A D, and opened out flat, the figure exhibited will be that of a *Sector*, whose area = $\frac{1}{2}$ arc × radius. Therefore, area of its curved surface = $\frac{1}{2}$ perimeter at base × length of slant side.

Case I.

	Base.	*Side.*	*Ans. Area.*
1.	14 yds.	42 ft.	98 yds.
2.	14 ft. 8 in.	4 ft. 8 in.	8 yds. 7 ft. 32 in.
3.	16$\frac{1}{2}$ ft.	2 ft. 11 in.	2 yds. 6 ft. [illegible]

	Base.	*Side.*	*Ans. Area.*
4.	19 yds. 1 ft. 8 in.	18 yds. 2 ft.	182 yds. 4 ft. 96 in.
5.	36 yds. 2 ft.	8 yds. 2 ft. 3 in.	160 yds. 3 ft. 108 in.
6.	15 ft. 4½ in.	6 yds. 1 ft. 7½ in.	16 yds. 6 ft. 132½ in.

Case II. Given, base, perimeter, and slant side: find height.

	Base.	*Slant.*	*Ans. Height.*
1.	2 ft. 9 in.	13 in.	11·892 in.
2.	8 ft. 3 in.	17½ ft.	5 yds. 2 ft. 5½ in.
3.	58 yds. 2 ft.	68 yds.	67 yds. 1 ft.
4.	15 ft. 7 in.	13½ ft.	4 yds. 1 ft. 3½ in.

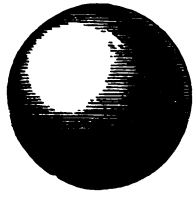
Case III. Given, base and height: find slant side.

	Base.	*Height.*	*Ans. Slant.*
1.	5 ft. 6 in.	9 ft. 8 in.	3 yds. 0 ft. 8½ in.
2.	15½ yds.	37 yds.	37½ yds.
3.	14 ft. 8 in.	32 ft.	10 yds. 2 ft. 1 in.
4.	7 ft. 9½ in.	19½ ft.	6 yds. 1 ft. 6½ in.

THE SPHERE.

1. If an orange be cut exactly in two, it may be conceived that the spherical surface of each half is double that of its flat side. Calculation and experiment have established this as a fact; and hence it follows that the spherical surface of the whole orange is *quadruple* that of this plane circle whose diameter is the same as that of the sphere.

Fig. 61.



$$\text{Now, area circle} = \frac{3\frac{1}{7}}{4}\ \text{diameter}^2.$$

$$\therefore \text{area sphere} = 4\left(\frac{3\frac{1}{7}}{4}\ \text{diameter}^2.\right)$$

$$= 3\tfrac{1}{7}\ \text{diameter}^2.$$

2. The spheroid may be considered as following the same law. (*Vid.* Spheroid, Solid.)

1. Find area of following spheres, whose diameters are respectively, $10\frac{1}{2}$ in.; 2 ft. 11 in.; 5 ft. 3 in.; $13\frac{1}{2}$ ft.; $17\frac{3}{4}$ yds.; 8120 miles.

2. Find area, &c., &c., whose circumferences are, 2 ft. 9 in.; 6 ft. 5 in.; 8 ft. 3 in.; $16\frac{1}{2}$ ft.; 37 yds.

Answers.

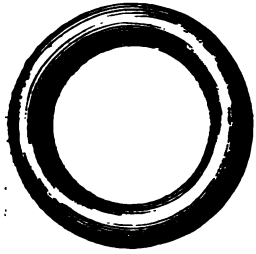
1. 2 ft. $58\frac{1}{2}$ in.; 2 yds. 8 ft. 106 in.; 9 yds. 5 ft. 90 in.; 63 yds. 5 ft. $113\frac{1}{4}$ in.; 980 yds. $8\frac{3}{4}$ ft.; 207222400 miles.

2. 2 ft. $58\frac{1}{2}$ in.; 14 yds. 3 ft. 58 in.; 2 yds. 3 ft. $94\frac{1}{4}$ in.; 95 yds. 0 ft. $92\frac{1}{2}$ in.; 429·0974 yds.

THE SOLID RING.

Case I. [*N.B.* The ring is considered as lying flat on a plane surface.]

Fig. 62.



1. Find upper surface of a rectangular ring, whose outer and inner diameters are 3 ft. 8 in. and 2 ft. 2 in.

Ans. 6 ft. 126 in.

2. Find upper and under surface of a rectangular ring, $1\frac{1}{2}$ yd. broad, and 13 ft. inner diameter. *Ans.* 55 yds.

3. Find total surface of a rectangular ring 6 in. thick, whose outer and inner diameters are 7 ft. 4 in. and 4 ft. 4 in.

Ans. 8 yds. 1 ft. 48 in.

4. Find do. do., 9 ft. broad and 3 ft. thick, inner diameter being 26 ft. *Ans.* 440 yds.

5. Find do. do., 6 yds. broad and 2 yds. thick, extreme diameter being 88 yds. *Ans.* $1546\frac{1}{2}$ yds.

Case II. When the ring is split horizontally.

1. The outer and inner diameters of a solid cylindrical ring are 3 ft. 8 in. and 2 ft. 2 in. It is split horizontally into 2 equal semicylindrical rings. Find area of the horizontal section. *Ans.* 6 ft. 126 in.

2. Do. do. 5 ft. 9 in. and 3 ft. 7 in.: find do. *Ans.* 1 yd. 6 ft. 128 in.

3. Do. do. 2 ft. $3\frac{1}{2}$ in. and 1 ft. $7\frac{1}{2}$ in.: find do. *Ans.* 2 ft. $7\frac{1}{2}$ in.

4. Do. do. 49 ft. and 28 ft.: find total surface of each half.

Ans. 3267 ft.

5. Do. do. 8 ft. 2 in. and 4 ft. 8 in.: find ditto. *Ans.* 90 ft. 108 in.

6. Inner diameter of do. is 4 ft. 5 in., and circumference of the cylinder which forms it is 2 ft. 9 in.: find cost of painting total surface when split horizontally into 2 equal parts, at 1*s.* 8½*d.* per sq. yd.

Ans. 14*s.* 0¾*d.*

MISCELLANEOUS QUESTIONS ON THE DIFFERENT SOLIDS.

1. Rails for a line of 29½ miles weigh 4867½ tons: find cost for 17 miles, iron being ¾*d.* a lb. *Ans.* £19365.

2. If this last metal were cast into a solid rectangular prism, standing on a base 2 yds. square, how high would it reach? (*Vid.* Tables.)

Ans. 118 yds. 0 ft. 2½ in.

3. A cask of treacle cost £3 5*s.* 4*d.* If it had weighed 2½ cwt. it would have cost £4 1*s.* 8*d.*: find (1.) weight, (2.) price per lb., (3) advanced price per lb. at which it must be sold, so as to realise a profit of 14*s.* on the whole. *Ans.* 2 cwt.; 3¼*d.*; 4¼*d.*

4. 17 planks, 23 inches wide at one end, and 17 inches at the other, are cut out of a tree 55½ ft. long: find value of the lot at 1*s.* 6*d.* a sq. ft.

Ans. £29 9*s.* 8½*d.*

5. A pit is $(13\frac{3}{4} \times 17\frac{1}{8} \times 29\frac{7}{16})$ ft.: find value of its contents at 6*s.* 2½*d.* a cubic yd. *Ans.* £81 14*s.* 10½*d.*

6. Find value of 27¾ stacks of hay, each containing 27¾ tons, at 3¾ gs. per ton. *Ans.* £3032 2*s.* 5$\frac{1}{16}$*d.*

7. A cubic yd. of coal weighs about a ton: find value of a seam (horizontal bed, layer, or stratum) 5¼ ft. thick, which extends under the whole of an estate of 376¼ acres, (allowing a deduction of ⅓ for waste, &c.) at 7½*d.* a cwt. *Ans.* £1594477 10*s.*

8. If the seam be 7 ft. thick, find the most compact rectangular area that must be excavated, in order to obtain 245000 tons.

Ans. 324·037 yds. sq.

9. Of how many different integral dimensions may a cistern be constructed, so as always to contain 60 cubic yds. of water? And which will be most compact?

Ans. { 10, viz. (3 × 4 × 5). { (1 × 1 × 60); (1 × 2 × 30); (1 × 3 × 20); &c. &c. }

10. How many persons could be supplied with a pint each out of this cistern? *Ans.* 80847, and $\frac{32}{277}$ pts. over.

11. A perfectly cubical block of stone stands 6 ft. high, and weighs 2634 oz. per cubic ft.: find weight and value at 1*s.* 2*d.* per cwt.

Ans. { 15 tons, 19 cwt. 3 qr. 17 lbs. £18 13*s.* 2$\frac{3}{4}$*d.*

12. 20 yds. of bar iron $1\frac{1}{4}$ inch sq. weigh $1\frac{1}{2}$ ton: find value of 50 ft. do. $\frac{7}{8}$ inch sq. at $1\frac{1}{2}d.$ a lb. *Ans.* £10 0*s.* 1*d.*

13. A bastion on the edge of a moat is 79 ft. high, and a cord stretched from the top to opposite edge is 97 ft.: find width of moat. *Ans.* 56·284 ft.

14. A quarry measures $(23\frac{1}{4} \times 25\frac{1}{2} \times 27\frac{3}{4})$ ft.: find value of the stone extracted at 1*s.* 3*d.* a cub. ft. *Ans.* £1028 5*s.* $4\frac{7}{33}d.$

15. If this stone be formed into a square prism 5 yds. high, what is the most compact area upon which it can stand? *Ans.* 33·1182 ft. sq.

16. A building $(96 \times 38\frac{1}{2})$ ft. has its foundations laid 17 ft. deep. Find cost of excavation at 1*s.* $3\frac{3}{4}d.$ a cub. yd. *Ans.* £152 14*s.* 4*d.*

17. $14\frac{3}{4}$ miles water piping weigh 9735 tons. Find weight of 3 furlongs 5 poles, and cost of whole at 4*s.* 8*d.* per cwt.

Ans. 257 tons. $16\frac{1}{4}$ cwt.; £45430.

18. If all this iron ($14\frac{3}{4}$ miles) were cast into a solid cylinder, 44 ft. round, what would be its height? *Ans.* 97 yds. 0 ft. $4\frac{3}{10}$ in.

19. How much higher would it be if it were cast with a hollow centre, $10\frac{1}{2}$ in. diameter. *Ans.* $13\frac{1}{7}$ in.

20. Part of a ship's cargo consists of 59 crates, containing each $13\frac{3}{4}$ sets of china, worth £3 17*s.* 6*d.* a set; 37 ditto, each $9\frac{1}{2}$ sets at £4 9*s.* $4\frac{1}{2}d.$; and 29 ditto, each $11\frac{1}{2}$ sets, at £2 16*s.* $10\frac{1}{2}d.$ Find value of this consignment. *Ans.* £5662 15*s.*

21. If each of the above crates be $(7\frac{1}{2} \times 5\frac{1}{2} \times 4)$ ft., find depth of the ship's hold which is filled with them, being $37\frac{1}{2}$ ft. long and $27\frac{1}{2}$ ft. wide. *Ans.* 20 ft.

22. A reservoir is (26 × 19) ft. surface, with perpendicular sides. How much water must be drawn off to sink it 18 in. (*Vid.* Tables.)

Ans. $4622\frac{2}{3}$ gals.

23. A cistern is (3 ft. 2 in. × 2 ft. 8 in.) and $2\frac{1}{2}$ ft. deep. Find cost of lining it with lead, 7 lbs. to the ft., at 42 gs. a ton.

Ans. £4 18*s.* $8\frac{3}{4}d.$

24. A rectangular log of mahogany is $30\frac{1}{2}$ ft. long, and measures (14 × 12) in. at one end, (6 × 4) in. at the other. Find its value at 2*s.* 3*d.* cub. ft. *Ans.* £2 0*s.* 2*d.*

25. A square pyramid stands on 97535376 sq. in., and is 279 ft. high. Find cost of facing its sides with plaster at $13\frac{1}{2}d.$ a sq. yd.

Ans. £5114 11*s.* $8\frac{1}{2}d.$

26. Books are (9 × 6) in. and $\frac{3}{8}$ in. thick. What must be the depth of a box $(2\frac{1}{4} \times 1\frac{1}{2})$ ft. inside, to contain 360 vols.? *Ans.* 15 in.

27. How many will the box hold if they are $\frac{5}{8}$ in. thick. *Ans.* 216.

28. The wood being $\frac{1}{2}$ in. thick, find difference between external and internal areas. *Ans.* 1 ft. 102 in.

29. A bucket is $10\frac{1}{2}$ in. diameter, and 16 in. deep. How much heavier is it when full of water than empty? *Ans.* 50 lbs. $2\frac{1}{12}$ oz.

30. There are 3 other buckets of the same depth, which hold respectively 2, 3, and 5 times as much water. Find diameter of each.

Ans. 14·847 ; 18·186 ; 23·2995 in.

31. If the diameter of the bucket be doubled while the depth is halved, what change will be made in its capacity ? *Ans.* Doubled.

32. A square pyramid 20 yds. high and 30 yds. along one side of base is faced with brick at 5*s.* 6*d.* a yd. square. Find cost.

Ans. £412 10*s.*

33. Tap *A* fills a cistern in 3 hours, and *B* in 4 hours, while *C* could empty it in 2 hours. (1.) How long will it be before the cistern is full, when all are open together ? (2.) How much less time will it take to fill it, if *B* is open an hour before the other two ?

Ans. 12 hrs. ; 2 hrs.

34. Two cylinders, each 12 ft. high, contain 225 and 900 cub. in. Find interior surface of each. *Ans.* 637 and 1274 sq. in.

35. *A*, *B*, *C*, *D* are men of war. *A* has 130 guns ; *B* 120 ; *C* 90 ; *D* 74. One third the whole number of guns throw 18 lb. shot ; half the remainder 42 lbs. ; and the rest 68 lbs. In an action lasting 1½ hours, each gun is fired on an average 5 times in 9 minutes. Find weight of metal discharged, and cost at £7 per ton. *Ans.* 394⅔ tons ; £2760.

36. If this metal could be formed into a single solid ball, what would be its diameter ? *Ans.* 5 yds. 0 ft. 1⅔ in.

37. A cistern's dimensions are (6 × 5 × 4) ft. It is required to enlarge all 3 sides in the same proportion, so that it may hold just twice as much. Find new dimensions. (*Vid.* Solid, Notes 10—12.)

Ans. 7·5594 ; 6·2995 ; 5·0396 ft.

38. Gin, worth 2*s.* 6*d.* a pint, is mixed with gin worth only 1*s.* 6*d.* a pint, so that a pint of mixture is sold without loss for 1*s.* 9*d.*: find the quantity of each in an 8 gallon anker. *Ans.* 2 and 6.

39. Two idle fellows having begged a gallon of beer at a tavern, the landlord thought to put them off by saying that they should have it if they could divide it equally with only a 5 pint and 3 pint jug besides the gallon bottle which contained it. Much to his dissatisfaction they contrived a way of doing it exactly with these three measures. Can *you* ?

40. A vat of 120 gallons is filled in 10 minutes by taps *A* and *B*, which run *successively*. *A* discharges 14 gals. per min., *B* 9 gals. How long does each run ? *Ans.* *A*, 6 ; *B*, 4.

41. Small beer and ale are mixed. 25 gals. more than half is beer, and 5 gals. less than a third is ale. The mixture is drawn off into ten casks : find quantity of beer and ale in each. *Ans.* 8½ and 3½ gals.

42. Two circular courts measure 40 and 50 feet diameter. Paving of latter costs £74 16*s.* 8*d.* : find cost of former at same rate.

Ans. £47 17*s.* 10½*d.*

43. A brewer supplied ale, worth 1*s.* 4*d.* a gallon, to two publicans, through pipes which discharged 13 gals. a minute each, the supply each time being as much as would run in half an hour. After a while *A* ordered twice and *B* thrice as much. Instead of letting these pipes run twice and thrice as long, the brewer thought to effect his object and save time by putting to his vat two new pipes of double and treble diameter, and running them half an hour as before. On the first occasion of using them, however, he found to his astonishment that in 15 minutes the vat was empty. How much would he have lost by this mode of fulfilling his contract, had they continued to run the whole time? *Ans.* £208.

44. As it was, how much more was delivered to each than the quantity paid for as twice and thrice the original supply? And what did the vat hold? *Ans.* { *A* right; *B* 585 gals. / Vat 2535 gals.

45. How long should each of the new pipes run to give exactly the same quantity as would have been discharged by the old ones running twice and thrice the time respectively? *Ans* 21′ 12·78″; 17′ 19·26″.

46. The vat being 3 ft. deep, what must be its diameter to hold the exact quantity required at one time by his two customers? *Ans.* 11½ ft.

47. An island ¾ mile square is covered with guano to an average depth of 14 ft. At 3¾ gs. a ton, what is its value, reckoning 1½ cub. yd. to the ton? *Ans.* £21,344,400.

48. How long will the supply last, if 13 vessels weekly each take away an average cargo of 875 tons? *Ans.* 9 yrs. 8½ wks.

49. Supposing a cub. yd. of guano to weigh 11¾ cwt., what breadth must be given to a rectangular block 40 yds. long, so that 2350 tons may be extracted from it? *Ans.* 21 yds. 1⅞ ft.

50. The parapet of a battery is 47 yds. long, 8 yds. thick, and 7 ft. high; and is composed of gabions containing each ⅔ cub. yd. of earth: how many are employed? *Ans.* 1316.

51. There are two buildings (60 × 45) ft. and (96 × 38½) ft. Foundations are 16 and 17 ft. deep. The former excavation costs 1*s.* 3*d.*, and the latter 1*s.* 3½*d.* per cub. yd.: find expence of each.

Ans. £100; £152 14*s.* 4*d.*

52. A solid yard of goods weighing 18 cwt., find the tonnage capable of conveyance in a ship's hold measuring (48½ × 27½ × 23) ft.

Ans. 1022 tons, 10 cwt. 3 qrs. 9½ lbs.

53. A square pyramid covers 1327104 sq. inches, and is 165 ft. high: find cost of facing with brick at 7*s.* 6*d.* a sq. yd.; also find length of its angular edge, *i. e.* from apex to angle at base.

Ans. £1374 14*s.* 4¾*d.*; 178·418 ft.

54. Diameter of circular spire at base is 12 yds., and length of slope 45 yds. Find expence of covering it with lead at 1*s.* 6*d.* a square foot.

Ans. £572 1*s.*

55. A cylinder is $17\frac{1}{2}$ inches diameter and $4\frac{3}{4}$ ft. high: what weight of water will it contain? *Ans.* 4 cwt. 1 qr. 20 lbs. $1\frac{1693}{1728}$ oz.

56. 150 gals. water flow in 4 seconds through a pipe $1\frac{3}{4}$ inches diameter. How much will flow through a $5\frac{1}{4}$ inch pipe at same rate in $7\frac{3}{4}$ hrs.? *Ans.* 9416250 gals.

57. Bricks, measuring $(9 \times 4 \times 3)$ inch, and worth 20*s.* 10*d.* a thousand, are stacked in a rectangular mass $13\frac{3}{4}$ ft. high, and cover a space 6 yds. square. Find value of stack. *Ans.* £74 5*s.*

58. A cupola is $\frac{3}{4}$ of a sphere which is 18 yds. 1 ft. in circumference. What will it cost to cover with lead, 7 lbs. to the foot, at 20 gs. a ton. *Ans.* £47 7*s.* $5\frac{1}{2}$*d.*

59. A piece of ground $(28\frac{1}{2} \times 19\frac{3}{4})$ yds. is opened for a quarry to a depth of 31 ft. from the surface, 7 ft. of gravel, &c. being removed before reaching the stone. Find value of stone extracted, at 1*s.* $4\frac{3}{4}$*d.* a cub. ft. *Ans.* £8485 6*s.* $9\frac{3}{4}$*d.*

60. A roller, $3\frac{1}{2}$ ft. in diameter and $3\frac{1}{2}$ ft. wide, makes 17 revolutions in passing from one end of a lawn to the other. Find area rolled in 13 journeys up and down. *Ans.* 1890 yds. 7 ft.

61. A tank (26×19) ft. has its surface lowered 18 inches by a pipe which discharges 5 cub. ft. in 3″. How long is it running? *Ans.* 5′ $17\frac{1}{7}$″.

62. From a cask which is $\frac{1}{3}$ full, 2 gals. are drawn, and then $\frac{3}{5}$ remainder, when it is $\frac{1}{6}$ full. Find capacity of cask. *Ans.* 36 gals.

63. From a tub of spirits containing 25 quarts, 5 qts. were drawn and 5 qts. water substituted, after which 15 qts. of the mixture were removed. The spirit being worth 18*s.* a gallon, find value of the mixture left. *Ans.* 36*s.*

64. A cask has 6 gals. ale at 6*d.* a qt. and 6 gals. at 8*d.* a qt. put into it., and then is filled up with beer at 4*d.*, so that the whole is worth 5*d.* a qt. Find capacity of cask. *Ans.* 36 gals.

65. Find cost of gilding a ball 6 ft. diameter at $3\frac{1}{2}$*d.* square inch. *Ans.* £237 10*s.* nearly.

66. A tank is $(7 \times 4\frac{1}{2})$ ft. and a yard deep. It is required to enlarge all its dimensions proportionally, so as to hold thrice as much. What addition will be made to each? *Ans.* 3·0597 ; 1·99 ; 1·3267 ft.

67. An 8 ft. wide trench is dug on every side of a piece of ground (300×200) ft., which is raised 1 ft. by the excavated earth spread over it. How deep is the trench? *Ans.* $7\frac{33}{38}$ ft.

68. Find the calibre, *i.e.* diameter of bore, of 4 guns which throw respectively 32, 24, 18, and 9 lbs. solid shot. *Ans.* 6·41 ; 5·82 ; 5·29 ; 4·2 in.

69. Find weight of a hollow 13-in. shell, iron being $1\frac{1}{4}$ inch thick. *Ans.* $84\frac{3}{4}$ lbs.

70. *A*, *B*, *C* are 3 regiments, numbering 789, 876, and 945 men. In an

action lasting 2 hours, $\frac{1}{2}$ the whole number fire twice in 5′, $\frac{3}{5}$ fire thrice in 8′, and the rest 7 times in 24′. Find value of the shot and powder consumed, balls being 24 to the lb., and powder $\frac{1}{4}$ weight of shot; latter being 80 gs. a ton, and former 30 gs.

Ans. Pr. £42 8*s.* 3*d.*; Lead, £63 12*s.* 4$\frac{1}{2}$*d.*

71. How much gilding will be required for the leaves of a book (18 × 13) in. and 2$\frac{3}{4}$ in. thick? *Ans.* 121 in.

72. $\frac{3}{4}$ in. planks, 27$\frac{1}{2}$ ft. long, and 11 in. wide, are laid close together, so as to form a perfect cube of 35937000 solid inches. How many planks are there? *Ans.* 1320.

73. How many such planks would form a *hollow* cube to contain the above solidity? *Ans.* 180.

74. The girth of a tree at its root is 16$\frac{1}{2}$ ft., and at a height of 24 yds. is 8$\frac{1}{4}$ ft. Find value at 9*d.* a cub. foot. *Ans.* £32 17*s.* 9$\frac{3}{4}$*d.*

75. A conical spire is 66 ft. in circumference at base, and 50 ft. high. Find cost of covering it with lead, 9 lbs. to the ft., at 20 gs. a ton. *Ans.* £142 5*s.* 1$\frac{1}{2}$*d.*

76. Diameter of a conical tent at base is twice its height, which is 10 ft. 6 in. Find the sum at which 1500 such should be contracted for canvas being 3*s.* 6*d.* a square yard. *Ans.* £14292 3*s.* 3*d.*

77. From a vat which was $\frac{3}{5}$ full, 35 gals. were drawn, and it was then $\frac{3}{8}$ full: find its capacity. *Ans.* 120 gals.

78. A tub of spirits, containing 5$\frac{1}{2}$ gals. worth 25*s.* a gal., was filled up with water, and the mixture sold without loss at a reduction of 25 per cent.: find tub's capacity. *Ans.* 6 gals. 7 pts.

79. Into a cask containing 12 gals. of water is put a quantity of whisky worth 24*s.* a gallon, and the mixture sold at 10*s.* a gal., with a gain of 25 per cent.: find cask's capacity. *Ans.* 18 gals.

80. A football, 22 inches circumference, is to have a second coat of leather: how much will be wanted? *Ans.* 154 inches.

81. A snowball is just a yard in diameter: how many small ones, 5$\frac{1}{2}$ inches circumference, can be made out of it? *Ans.* 8705$\frac{1}{2}$.

82. Arrange the 9 digits in a solid square, so that the sum of the 3 figures in any row shall always be 15, whether reckoned horizontally, vertically, or diagonally.

83. Show how to plant (1.) 4 trees at exactly equal distances from each other, (2.) 9 do. so that 3 may be seen in a line or row in 10 different ways, *i. e.* reckoning no row of 3 twice over.

84. A circular measure is 18 inches diameter, and holds 2 bushels of seed: find depth. *Ans.* 17$\frac{1}{2}$ in. nearly.

85. If the depth be doubled, how much must diameter be increased so that it may hold (1.) 8, (2.) 6, (3.) 4 times as much as now?

Ans. 18; 13·185; 7·455 in.

86. If the seed be heaped up into a cone having a slope of 45°, what will be the value of the measure at 2*s.* 6*d.* a gal.? *Ans.* £2 6*s.* 10½*d.*

87. The conical cap of a tower is 12 yds. diameter at base and 24 ft. high: find expence of leading it at 13*s.* 6*d.* a sq. yd.

Ans. £127 5*s.* 8½*d.*

88. A conical spire is 44·5982 yds. high, and slope 45 yds.: find cost of leading it at 1*s.* 6*d.* a sq. ft. *Ans.* 572 15*s.* 8½*d.*

89. At 13*s.* 6*d.* a sq. yd., find value of a glass aquarium ($6\frac{1}{3} \times 4$) ft. and 20 inches deep, and also quantity of water in it when full.

Ans. £4 11*s.* 6*d.*; $270\frac{199}{277}$ gals.

90. From a cask $\frac{5}{8}$ full, $\frac{1}{3}$ was drawn, and then $\frac{1}{4}$ the remainder. Tasting consumed $\frac{1}{10}$ this second remainder, and $\frac{1}{3}$ of what was left leaked away, leaving 3 gals. in the cask: find its whole capacity.

Ans. 9 gals.

91. A railway cutting is 29 ft. wide at bottom and 83 ft. at top, slopes being uniform all the way. Its ⊥ depth is 32 ft. How many cubic yds. of earth are excavated in ¼ mile? (*Vid.* Trapezoid.)

Ans. 87608⅓ yds.

92. A garden (240 × 170) ft. has a walk 5 ft. wide all round. To what depth must a circular gravel pit, $10\frac{1}{2}$ ft. diameter, be excavated to cover the whole walk 3 inches deep? *Ans.* $11\frac{199}{231}$ ft.

93. Find length of diagonal of a cube whose faces contain 361 square inches each. *Ans.* 26·87 in.

94. A trough is $5\frac{3}{4}$ ft. long, 17 in. wide, and 14 in. deep: find expence of painting it inside and out at 1*s.* 8½*d.* a sq. yd. *Ans.* 9*s.* 4*d.*

95. A chest, 43 inches wide and 38 inches high, stands on a space of $23\frac{7}{11}$ sq. ft.: find length, breadth, and diagonal of a board which fits into it diagonally, and divides it into 2 equal parts, (1.) when board stands ⊥ to base, (2.) when it slopes from back to front.

Ans. { (1.) 89·067; 38; 96·8349 in.
(2.) 78; 57·384; 96·8349 in.

96. A circular conduit, 1 yd. diameter, empties a pond in 20°: find size of another that will fill it again in 4°. *Ans.* 6·7082 ft.

97. A 1½ in. pipe discharges 157 gals. per minute. What quantity will flow in $4\frac{1}{2}$ hours through another of 3 in. diameter?

Ans. 169560 gals.

98. What must be the calibre (*Vid.* Ques. 68.) of two guns which throw 42 and 68 lbs. solid shot? *Ans.* 6·58 in. and 7·728 in.

99. Ale worth 15*s.* a barrel is mixed with beer at 9*s.* ditto, so that a 9 gal. cask of the mixture is worth 10*s.* 6*d.* What quantity of each does it contain? *Ans.* 2¼ at 15*s.* &c.

100. Beer at 3*d.* a quart is mixed with ale at 5*d.*, so that a gallon cask of the mixture is worth 12*s.* Find quantity of each sort in the cask. *Ans.* 7½ and 1½ gals.

101. An exactly cubical box contains 2 yds. 14 ft. 145 in. solid matter. Find cost of covering it all over with oilskin at 1s. 8½d. a yd. (27 in. wide) and of banding all the edges with brass at 6d. a ft.

Ans. £1 5s. 0½d.; 13s. 4d.

102. A parapet 37½ yds. long, 8 yds. thick, and 7½ ft. high, consists of sand-bags and fascines. $\frac{7}{11}$ of the mass are the latter, and 12 bags contain 15 ft. of earth. How many are used? *Ans.* 13680.

103. Ditto, ditto is (49½ × 8½) yds. and 8 ft. high, and consists of sand-bags, each containing $\frac{3}{11}$ yds. of earth. How many are used, and how many men employed to carry them, if each party of 13 carry 18 amongst them? *Ans.* 20196 bags and 14586 men.

104. A rectangular ring of elliptic shape has its extreme major and minor axes 35 and 21 yds., while breadth and thickness of its substance are 3½ and 2½ ft. Find total surface. *Ans.* 337½ yds.

105. A solid cylindrical ring of elliptic shape is split horizontally into 2 equal parts. Its extreme major and minor axes are 4 ft. 1 in. and 2 ft. 11 in., while diameter of ring is 3½ in. Find total surface of each half. *Ans.* 1089 in.

106. A tank is to hold 5000 gals., and to be 11 ft. long. Its width being 8 ft., what must be its depth? *Ans.* 9 ft. $1\frac{462}{1331}$ in.

107. Reckoning this depth as 9 ft. 1¼ in., and allowing 11 lbs. lead to the foot, find cost of tank, at 40 gs. a ton, tank being open at top.

Ans. £89 10s.

108. If this lead were formed into a solid cone, 21 inches diameter at base, find its height, and area of curved surface.

Ans. { 33·396 in. height. 1155 in. area.

109. Find cost of completely covering with tin, at 3s. 4½d. sq. yd., a box measuring 3 ft. along each edge. *Ans.* 20s. 3d.

110. A lb. of powder occupies 32 cub. in. What will be the expence of lining with copper 13½ oz. to the foot, and 2s. a lb., a perfectly cubical box to contain 128 lbs.? *Ans.* 18s.

111. A chest measures (5½ × 4⅔ × 2¾) ft. Find cost of painting it all over at 1s. 8½d. a sq. yd., and banding every edge with tin at 2½d. a foot.

Ans. 20s. $1\frac{5}{16}$d.; 9s. 8½d.

112. Painting a trough at 1s. 6d. a sq. yd. cost 9s. 6d. Its length was 8½ ft. and depth 21 inches. Find breadth. *Ans.* 3·83 ft.

113. A railway cutting is 38 ft. wide at bottom and 74 ft. at top, 35 ft. deep, and ¾ mile long. Slopes are uniform, and dimensions the same all the way. How many solid yards of earth were removed?

Ans. 95822½.

114. This earth just filled up a circular pit with ⊥ sides 96 yds. deep. What was its circumference? *Ans.* 112 yds.

115. 250 gals. water flow in 15″ through a 2¾ inch pipe (i. e. in the

meter). Find quantity flowing at same rate through 2 others of $5\frac{1}{2}$ and $8\frac{1}{4}$ inch diameter in 3° 45′. *Ans.* 900000; 2025000 gals.

116. A square pyramid occupies an area of 8088336 inches, and is 97 yards high. Find (1) volume, (2) whole surface area, (3) length of angular edge. *Ans.* 201792$\frac{1}{2}$ cub. yds.; 16548 sq. yds.; 111·933 yds.

117. The parapet of a battery was $45\frac{1}{2}$ yds. long and $6\frac{1}{2}$ yds. thick, and contained 18632$\frac{1}{2}$ cub. ft. How deep was the trench? *Ans.* 7 ft.

118. A parapet $(48 \times 8\frac{1}{2})$ yds. and $7\frac{1}{2}$ ft. high is constructed of gabions holding each $\frac{3}{4}$ cub. yd. How many are used? *Ans.* 1836.

119. If such a parapet were built of fascines and sand-bags, the former occupying $\frac{7}{11}$ of the whole mass, and each bag holding 2$\frac{1}{2}$ cub. ft. earth, how many bags would be wanted? *Ans.* 11186.

120. A 13-inch shell made of $1\frac{1}{4}$ inch iron is filled with powder. Find value of shell, iron being 7*l.* a ton and powder 8*d.* a lb.

Ans. 22*s.* 2*d.*

121. *A*, *B*, *C*, are three batteries, *A* being armed with twenty-six 24-pounders, *B* twenty-seven 36-pounders, and *C* twenty-nine 42's. The general orders a cannonade of 4° 45′, during which each gun is to fire 4 times in 19′, with charges of powder $\frac{1}{4}$ wt. of shot. Find (1), number of shot and pounds powder to be provided by the captain of each battery; (2) expence of action, iron being £4 13*s.* 4*d.* a ton, powder 12*s.* 3*d.* a stone.

Ans. { *A*, shot 1560, powder 9360 lbs.
B, „ 1620, „ 14580 „
C, „ 1740, „ 18270 „
Expence £2198 8*s.* 9*d.*

122. A canal, $17\frac{1}{3}$ miles long, is 30 ft. wide at top, 18 ft. wide at bottom, and $1\frac{1}{2}$ yds. deep. How soon would the excavation be completed by 800 men, each removing, on the average, 15 cub. yds. per day?

Ans. 30$\frac{1}{2}$ days.

123. A railway embankment is 28 ft. wide at top, 11 yds. high, and 18 yds. wide at bottom. What length of it might be formed of the earth removed from the canal? *Ans.* 2458·536 yds.

124. A tank measures (13×11) ft. at top, (11×9) ft. at bottom, and 1 yd. in slope: find cost of cementing sides and bottom at 1*s.* 10$\frac{1}{2}$*d.* a sq. yd. *Ans.* £2 8*s.* 1$\frac{1}{2}$*d.*

125. A square pyramid standing on an area of 543169 inches, has its summit removed parallel to base (thus becoming a frustum), so that top and bottom sides are as 5 : 11, and a line joining them at right ∠s to both is 64 ft.: find length of slope at each corner. *Ans.* 66 ft. 1·86 *in.*

126. On sounding a well, $5\frac{1}{2}$ ft. diameter, 3 yds. of water were found in it: how many gals. were there? *Ans.* 1215$\frac{1}{4}$ nearly.

127. In 12° a pipe fills a circular pond 19 ft. diameter and 4 ft. deep: how soon will a pipe of thrice the calibre fill a reservoir 38 ft. diameter and 2 yds. deep? *Ans.* 8°.

128. At what rate (of gals. per minute) will the water flow through each pipe? *Ans.* 9·8302 and 88·472 gals.

129. And at what speed per hour, supposing the diameters to be 2 and 6 inches? *Ans.* 1444 yds. per hr. each.

130. Water enters a conduit 3 ft. diameter at rate of $5\frac{1}{2}$ miles per hr. and escapes from it where its diameter is only 18 inches: what is its velocity? *Ans.* $30\frac{1}{4}$ miles per hr.

131. Iron and lead piping are both 2 inches diameter inside, and $\frac{5}{16}$ in. thick: find wt. of 20 yds. of each. *Ans.* { 4 cwt. 0 qrs. 11 lbs. 15 oz. / 6 cwt. 0 qrs. 2 lbs. 9 oz.

132. A trough is 5 ft. 10 in. long and 17 in. wide. Cost of painting it inside and out at 1*s.* $8\frac{1}{4}$*d.* a sq. yd. is 8*s.* 1*d.* How deep is it? *Ans.* 11 inches.

133. In an elliptic lawn, whose major and minor axes are 91 and 63 yds., is formed a circular fish pond, diameter $31\frac{1}{2}$ ft. How much turf is left? *Ans.* $4571\frac{7}{8}$ yds.

134. And how much, if a gravel walk $3\frac{1}{2}$ ft. wide encircles the pond? *Ans.* $4529\frac{7}{72}$ yds.

135. Compare the area of an ellipse (35×21) yds. with those of the ⊙s described about its major and minor axes as diameters. *Ans.* 616; $962\frac{1}{2}$; $346\frac{1}{2}$ yds.

136. At 1*s.* $3\frac{1}{4}$*d.* per cubic yd., the sum of £152 14*s.* 4*d.* was paid for digging the foundations of a building $(96 \times 38\frac{1}{2})$ ft. How deep was the excavation? *Ans.* 17 ft.

137. A box which contains 6912 cubic in. covers 4 sq. ft.: find cost of leathering it all over at 3*s.* $6\frac{3}{4}$*d.* a sq. yd. *Ans.* 9*s.* 6*d.*

138. A breastwork 189 yds. long, 8 ft. thick, and $4\frac{1}{2}$ ft. high, is thrown up on level ground. Each sapper has a space of 7 ft. long to work in, and does $4\frac{1}{2}$ cubic yds. in 3°. How soon is the necessary excavation completed? *Ans.* 6° 13′ 20″.

139. Allowing 4 parapet builders to each sapper, how many men are engaged at one time? *Ans.* 405.

140. A parapet consists of $\frac{1}{4}$ gabions, $\frac{1}{3}$ fascines, and the rest sand-bags, containing each $\frac{1}{2}$ cubic ft. Dimensions of parapet are (22×6) yds. and $7\frac{1}{2}$ ft. high: find number of sand-bags required. *Ans.* 7425.

141. In 100 lbs. gunpowder, 75 lbs. are nitre, 15 charcoal, sulphur 10. Allowing 30 cubic in. to the lb., find the respective quantities of these 3 in a box $(20 \times 18 \times 15)$ inches inside, and value at 10*s.* 6*d.* a stone. *Ans.* { N. 135; C. 27; S. 18 lbs. / £6 15*s.*

142. *A*, *B*, *C*, *D*, *E*, are 5 batteries, armed as follows: *A* has 23 48-pounders; *B* 19, 68's; *C* 36, 32's; *D* and *E* each 13 mortars. In a bombardment of 5° 45′, each gun fires 3 times in 23′, with powder $\frac{1}{3}$ wt. of shot; while each mortar twice in 23′ throws a shell worth 3*l.* 4*s.*

with $2\frac{3}{4}$ stone powder: find cost of action, iron being £8 15*s.* a ton, powder 12*s.* 3*d.* a stone. *Ans.* £7337 2*s.* $2\frac{1}{4}$*d.*

143. Iron water piping is $10\frac{1}{2}$ inches diameter inside, and $\frac{3\frac{1}{2}}{8}$ inches thick. Giving the more exact weight of 4·526 oz. to a cubic inch, find value of $2\frac{3}{4}$ miles' piping at 10 gs. a ton. *Ans.* 3474 11*s.* $11\frac{1}{2}$*d.*

144. A steam cylinder is 8 ft. high and 91 in. outer diameter, metal being $\frac{7}{8}$ in. How much does the outer exceed the inner area? *Ans.* $3\frac{3}{4}$ sq. ft.

145. What weight of water will it contain, and what will be its value at £14 per ton? *Ans.* 9 tons, 14 cwt. 3 lb. $7\frac{1}{8}$ oz.; £42 1*s.* $3\frac{1}{2}$*d.*

146. A pit 55 ft. round and 96 yds. deep is full of water. How soon will it be emptied by an engine which pumps up 250 gals. per minute? *Ans.* 28° $49\frac{343}{1385}$'.

147. What must be the circumference of a reservoir 3 yds. deep to hold the water pumped up from the pit? *Ans.* 311·11 ft.

148. Diameter of a drum is $23\frac{1}{3}$ inches, and it stands 3 ft. 6 in. high: find cost of parchment at 6*s.* 9*d.* a sq. yd., and of material for the cylinder at 3*s.* $4\frac{1}{2}$*d.* a sq. yd. *Ans.* 4*s.* 11*d.*; 8*s.* $5\frac{1}{16}$*d.*

149. The earth removed from a railway cutting ($\frac{7}{8}$ mile long, 28 ft. deep, 29 ft. wide at bottom, and 48 ft. at top) is used to form an embankment on another part of the line 37 ft. high, 26 ft. wide at top, and 56 ft. at bottom. How long is it? *Ans.* $1094\frac{1}{2}$ yds.

150. A publican, selling off, had 7 full casks, 7 half-full, and 7 empty. Three persons bought his stock, each taking away 7 casks, and each the same quantity of beer. How did they contrive the distribution?

(*N. B.* The casks were taken just as the publican left them.)

151. The frustum of a square pyramid covers a space of 278784 ft., and side at top = $\frac{13}{88}$ side at bottom, shortest distance between them being 126 ft.: find length of slope at corner and area of 4 faces.

Ans. 143·798 ft.; 16968 yds.

152. *A*, *B*, *C*, are boxes to be covered all over with canvas at 1*s.* $9\frac{1}{2}$*d.* a yd., 34 in. wide. *A* measures $(3 \times 2 \times 1\frac{1}{2})$ ft., *B* $(2\frac{1}{2} \times 2\frac{1}{4} \times 2)$ ft., *C* $(2\frac{1}{2} \times 3\frac{1}{4} \times 4)$ ft.: find cost of each. *Ans.* 5*s.* $7\frac{1}{2}$*d.*; 6*s.* $3\frac{5}{8}$*d.*; 12*s.* $11\frac{1}{2}$*d.*

153. 658 gabions, each holding $\frac{3}{4}$ cub. yd. earth, are prepared for making a parapet 8 yds. thick and 7 ft. high. How long will it be?

Ans. $23\frac{1}{4}$ yds.

154. 6840 sand-bags, each holding $2\frac{1}{2}$ cub. ft., are used for a parapet $37\frac{1}{2}$ yds. long and $7\frac{1}{2}$ ft. high, $\frac{7}{15}$ of which are fascines: find thickness.

Ans. 8 yds.

155. A square house stands on 331776 inches. Roof rises 13 ft. ⊥, at an ∠ of 45°, and forms a △ on 2 sides, and a trapezoid on the other two: find area of each, and length of slope at the angles.

Ans. { △ 441·2352 ft. / Trap. 643·468 ft. / Slope 22·516 ft.

156. A chimney 78 ft. high is 6 ft. sq. at bottom and 3 ft. sq. at top. There average 72 bricks to the sq. yd. on each face: find cost at 18*s.* $2\frac{3}{4}$*d.* a thousand. *Ans.* £10 4*s.* 9*d.*

157. The weight of water in a tank is 42 tons, 17 cwt. 16 lbs., and its dimensions are as 2, 3, 4: find them. *Ans.* 8, 12, 16 ft.

158. A well 53 ft. deep and $3\frac{1}{2}$ yds. diameter is dug at $13\frac{1}{2}$*d.* per cubic yd., and bricked round at $4\frac{1}{4}$*d.* a sq. ft.: find cost of well.

Ans. { Dig. £ 9 11*s.* $3\frac{9}{16}$*d.*
Brk. £30 19*s.* $5\frac{1}{4}$*d.*

158. (B) If the volume or capacity of the tank in Question 157. be diminished one half by a proportional diminution in the sides, what will be its new dimensions?

Ans. { 6·3496 ft.
9·5244 ft.
12·6992 ft.

158. (C) If a tank, holding twice as much, and having sides in same proportion, be cut proportionally so as to hold only $\frac{1}{2}$ its present quantity, how much must be removed from each dimension?

Ans. 3·089; 4·637; and 6·178 ft.

159. An octagonal spire is 52 ft. perimeter at base, and 29 yds. high; distance between centres of two opposite sides being 16 ft. Find cost of leading the spire at 2*s.* $8\frac{1}{2}$*d.* a sq. ft. *Ans.* £ 307 12*s.*

160. This lead (8 lbs. per foot) being cast into a cone $17\frac{1}{2}$ inches diameter, find its height and area of curved surface.

Ans. { 15·2567 yds.
11 yds. 5 ft. 130 in.

161. A globular glass for gold fish, $\frac{3}{4}$ of a sphere, diameter 21 inches, is filled with water. Find weight of contents. *Ans.* 131 lbs. $9\frac{5}{32}$ oz.

162. Find internal area of a box standing on a square base, in which this glass might be packed. *Ans.* $1543\frac{1}{2}$ inches.

163. *A* and *B* are two lengths of piping, 3 inches diameter at one end and 2 inches at the other. Water flowing 9 miles an hour enters *A* at large end and escapes at small, but enters *B* at *small* and escapes at *large*. At what different speeds does it flow from each?

Ans. $20\frac{1}{4}$ and 4 miles an hour.

164. A circular marl pit, 28 ft. diameter and 16 ft. deep, is dug in a 2 acre field, and its contents spread evenly over the remaining surface. How deep will it lie? *Ans.* 4·161 inches.

165. Water piping for $3\frac{3}{4}$ miles weighs $1\frac{3}{4}$ cwt. per yd. and costs 10 gs. a ton. This is found to give only half the requisite supply, and ∴ needs to be replaced by other piping of sufficient bore. Cost being in proportion to weight, how much unnecessary expence is incurred by *doubling* the diameter? *Ans.* £3552 2*s.* $10\frac{3}{4}$*d.*

166. Diameter having been 8 inches at first, what should be the new?

Ans. 11·31368 inches.

167. A tank is emptied in 1° 30′ by a 12-inch pipe. How soon will it be emptied by a 2 ft. and 3 ft. pipe running together? *Ans.* $6\frac{37}{43}$.

168. A reservoir is 40 yds. square at top and 4 yds. deep. The banks slope inwards at an $\angle$ of 45°. How many gallons does it contain? *Ans.* 883937½.

169. Hydrogen gas is 8 times lighter than common air, of which 1 cubic foot weighs $1\frac{1}{4}$ oz. How many persons, weighing each $11\frac{1}{2}$ stone, could ascend in a car weighing $1\frac{1}{2}$ cwt. attached to a balloon containing 25000 cub. ft. of gas? *Ans.* 9 in the car will almost balance it.

170. A block of stone forms the frustum of a square pyramid, whose perimeter at base is 28 ft., and 20 ft. at top, height being 12 yds. Find value at 40*s.* 6*d.* a solid yard. (*Vid.* Notes on "Frustum.")

Ans. £98 2*s.*

171. Diameters of a conical frustum of marble are $17\frac{1}{2}$ and $10\frac{1}{2}$ in. and height 24 ft. Find value at £6 1*s.* 6*d.* per cub. yd.

Ans. £5 17*s.* $10\frac{1}{2}$*d.*

172. Dimensions of a prismoid of rosewood at base are (19 × 14) inches and (17 × 15) inches at top, height being 23 inches. Find value at 2*s.* 3*d.* a cub. ft. *Ans.* 7*s.* $9\frac{39}{64}$*d.*

173. Allowing 6·604 oz. to the cub. inch of lead, at 2 gs. a cwt., how much is overcharged by a plumber who sends in a bill of £35 for 125 yds. of inch bore piping, whose extreme diameter is $1\frac{1}{2}$ in.?

Ans. 16*s.*

174. *A* and *B* are spheroids of copper, *oblate* and *prolate* respectively. *A's* major and minor axes are 63 and 28 inches; *B's*, $19\frac{1}{2}$ and $15\frac{3}{4}$ inches. Find value of each at £224 a ton.

Ans. { £1857 13*s.* $9\frac{3}{4}$*d. A.*
{ £80 17*s.* $2\frac{3}{8}$*d. B.*

175. The elliptic links of a chain consist of iron $2\frac{3}{4}$ inches round. Major and minor axes (extreme) of each link are $10\frac{1}{8}$ and $8\frac{1}{8}$ inches. Find length of chain with 864 such links, and value at 30 gs. a ton, reckoning 4·526 oz. to cub. inch. *Ans.* $240\frac{7}{288}$ yds.; £56 17*s.* $1\frac{1}{2}$*d.*

176. A railway cutting, 15 ft. wide, with $\perp$ sides, is made through a ridge $\frac{1}{8}$ mile long, which rises regularly on both sides, 1 ft. in 5, to the summit. How much rock is removed? *Ans.* 24200 yds.

177. A hemispherical punchbowl is 5 ft. 6 in. round the brim. How many persons may be helped from it twice in hemispherical glasses $1\frac{3}{4}$ inches diameter. *Ans.* 864.

178. A builder's bill. Find total amount.

$29\frac{3}{4}$ thousand bricks at 20*s.* 10*d.* per 1000.
$17\frac{3}{4}$ cub. yds. stone at 2*s.* 3*d.* per foot.
$27\frac{1}{2}$ loads timber at $7\frac{3}{4}$ gs. a load.
$17\frac{3}{4}$ cwt. mortar at 35*s.* per ton.
157 sq. ft. glass at 13*s.* 6*d.* per sq. yd.

$35\frac{3}{4}$ gross nails at 2*s.* 1*d.* per hundred.
Wages of 17 men for 9 months, at 2*s.* 6*d.* a day.
Sundries, £375 7*s.* $5\frac{1}{2}$*d.*

Ans. £1200.

[*N.B.* 13 wks. in 3 months, and 6 working days per week.]

179. A builder's bill. Find total amount.

$17\frac{3}{4}$ cub. yds. stone at 2*s.* $8\frac{3}{4}$*d.* per ft.
$25\frac{5}{8}$ thousand bricks at 2*s.* 1*d.* per hundred.
$32\frac{3}{4}$ cwt. mortar at 23*s.* 4*d.* per ton.
$27\frac{1}{4}$ loads timber at $2\frac{1}{2}$ gs. per load.
3253 sq. ft. plank at 3*s.* $6\frac{3}{4}$*d.* per yd.
$237\frac{1}{2}$ sq. yds. glass at $9\frac{1}{2}$*d.* per ft.
$27\frac{5}{8}$ gross nails at $3\frac{3}{4}$*d.* per doz.
Wages 29 men $23\frac{1}{2}$ wks. at 2*s.* $3\frac{1}{2}$*d.* a day.
Sundries £312 1*s.* $0\frac{1}{2}$*d.*

Ans. £1100

180. A builder's bill. Find total amount.

$29\frac{3}{4}$ thousand bricks at 22*s.* 6*d.* per 1000.
$23\frac{3}{4}$ cub. yds. stone at 2*s.* 5*d.* per ft.
$13\frac{3}{4}$ loads timber at $5\frac{3}{4}$ gs. per load.
$18\frac{3}{4}$ cwt. mortar at 31*s* 8*d.* per ton.
$43\frac{3}{4}$ sq. yds. glass at 9*d.* per ft.
$23\frac{3}{4}$ gross nails at 20*s.* 10*d.* per 1000.
$1\frac{3}{4}$ thousand screws at 9*s.* per gross.
$27\frac{3}{4}$ pints paint at 6*s.* 8*d.* per gal.
$17\frac{3}{4}$ doz. slates at 4*s.* 7*d.* a score.
$10\frac{3}{4}$ sq. yds. lead (8 lbs. to ft.) at £63 per ton.
Wages of 27 men and 5 boys for 9 months; men at 2*s.* 9*d.* and boys $7\frac{1}{2}$*d.* a day.
Sundries £350 1*s.* $11\frac{1}{4}$*d.*

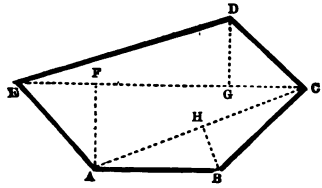
Ans. £1500.

APPENDIX.

IRREGULAR AREAS.

WHEN a field or other space is of irregular shape, as A B C D E,

Fig. 63.



its area may be found by dividing it into triangles, trapezoids, &c., whose separate areas can be calculated as *regular* figures, certain measurements being made (as E C, D G, A F, B H) in order to obtain base lines, perpendiculars, &c. Judgment must be exercised in this division, so as to have as few measurements as possible. For instance, if E C be parallel to A B, then A C and B H are needless, since E A B C is a trapezoid.

TO FIND A "MEAN PROPORTIONAL."

(*Vid.* Euclid, Bk. VI. 8, 13, 16.)

1. If a plate fall from the table to the floor, and break into four equal pieces, each piece is exactly a quarter of the whole. Now pick up three of these quarters and place

them on the table. Then the ratio or proportion (Euclid, B. v.) which the number picked up bears to the whole number on the floor is as 3 to 4,—usually written 3 : 4.

2. If this were written 3 ⁞ 4, it would correspond very closely with $\frac{3}{4}$ or $\frac{3}{4}$,—the usual mode of expressing this proportion in a fractional shape; where the horizontal line may be taken to represent the *table*, 4 the *name* (denominator) of the pieces on the floor; and 3, the *number* (numerator) picked up.

3. Now if each of these 4 "fractional parts" or broken pieces were to be equally broken in two, we should have 8 equal parts instead of 4, and each piece would be one *eighth* instead of one *fourth* of the whole.

4. If then we desire to replace on the table *as much* of the plate as before,—since the pieces are now *twice as small* (*i. e.* half as large), we must pick up *twice as many;* and the ratio or proportion of the picked-up pieces to the whole number will remain unaltered; *i. e.* the ratio of 6 : 8 is the same as that of 3 : 4.

5. Written in the ordinary way, this fact would be stated thus:—

$$6 : 8 :: 3 : 4,$$

which with a slight alteration would stand thus,

$$6 \vdots 8 :: 3 \vdots 4 \text{; and hence,}$$

$$\frac{6}{8} \text{ ..... } \frac{3}{4}, \text{ or, } \frac{6}{8} = \frac{3}{4}.$$

6. The truth of the above will be clearly seen if the thing "fractured" or broken be 1*l*. For each *fourth* will then be a crown, each *eighth* a half-crown; and it is obvious that 3 of the former and 6 of the latter must bear the same proportion to the whole.

7. If the above proportion be examined, it will be observed that the *same product* is obtained by multiplying together the two *outside terms* or "extremes," and the two

middle terms or "means." For $6 \times 4 = 24 = 8 \times 3$; *i. e.* "product of extremes = product of means." And no quantities are in true proportion which fail to give this result.

8. Let us now turn to account the principle thus established.

a. Find a 4th proportional to the numbers 6, 8, 9.

Let x represent the 4th proportional; then

$$6 : 8 :: 9 : x ; \therefore 6x = 72, \therefore x = 12.$$

And this is true; for as 6 is $\frac{3}{4}$ of 8, so 9 is $\frac{3}{4}$ of 12.

$$\text{And } 6 \times 12 = 72 = 8 \times 9.$$

[*N.B.* All "Rule of Three" questions should be done in this way.]

β. The ratio of 4 to a certain number is the same as the ratio of that number to 9. Find it.

Let x = the number required, then

$$4 : x :: x : 9 ; \therefore x^2 = 36, \text{ and } \therefore x = 6.$$

In this case 6 is the middle term or *mean*, and from its peculiar relation to both 4 and 9, it is called the "mean proportional" between them.

9. The "mean proportional," then, between any 2 quantities is found by *multiplying them together, and extracting the square root of their product.*

Ex. 1. A leg of mutton weighs 10 lbs. in one pan of a false balance, and only 5 in the other: find true weight. *Ans.* 7·071 lbs.

Ex. 2. The areas of the circles described about the major and minor axes of an ellipse as diameters are 52 and 117 yds.: find area of ellipse. *Ans.* 78 yds.

THE PROTRACTOR AND SCALE.

1. The Protractor or half-circle is usually made of brass or ivory, but may be readily constructed by any student for himself, of cardboard or other convenient material.

2. Its arc is divided into 180 degrees, as in *fig.* 65, counting from left to right inside, and from right to left

Fig. 64.

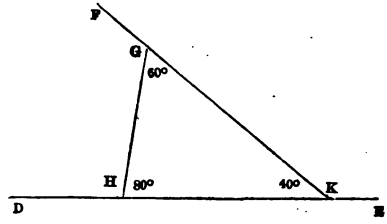
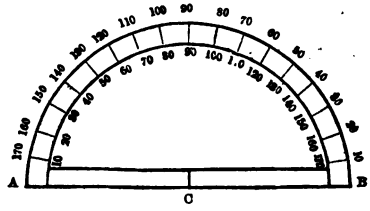


Fig. 65.



outside, or *vice versâ*. It has its diameter *A B* bisected in *C*.

3. Suppose it required to form an angle of 80° at H in line D E. Lay the protractor flat on the paper, so that A B coincides with D E, and C with H. Now look on the arc for 80 (reckoning always from 10 upwards), and put a dot on the paper at that place, G. Then *protract* these two points H and G into the line H G, and E H G is the ∠ required.

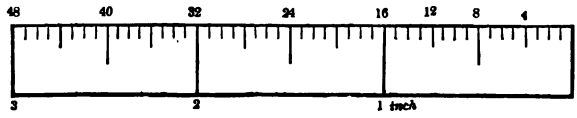
4. To form an ∠ of 60° at G, lay A B even with H G, and C upon G. Then, if the arc faces towards E, mark the point K in the direction where 60 appears upon it, and *protract* or join G K. H G K is the ∠ required.

5. If the arc had lain facing towards D, the angle would have been H G D instead of H G K.

6. If A B be laid even with D E, and C upon K, G K H and G K E will be found to be $\angle$s of 40° and 140° by measurement as well as calculation; and the same thing may be done at G and H.

THE SCALE OF EQUAL PARTS.

Fig. 66.



1. On a strip of cardboard mark off a length of 3 or more inches from a carpenter's rule, and divide each into 16 parts. This is the most convenient method, as it involves nothing but repeated division by 2. Each inch may thus be easily made to contain 32 or 64 equal parts, if desirable.

2. It is turned to account thus,—

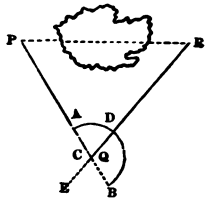
Suppose P Q and R Q in *fig.* 67. to be 38 and 49 poles, and $\angle$ P Q R = 50°. With compasses take off 38 and 49 small divisions from the scale, and draw lines of these lengths upon paper, using the protractor to get the $\angle$ P Q R. Then take the compass measurement of P R, and apply it to the scale, when its length will be found in poles.

3. If P Q and R Q are feet, P R will be feet too; and so of yards, miles, or any other unit of measurement.

THE THEODOLITE.

1. The Theodolite, as its name imports in Greek, is a "contrivance for observing" the angular distance between 2 or more objects.

Fig. 67.



2. Take a protractor, and at C, the middle of its diameter, fix a short screw with a ring in its head. Through this insert firmly a tin tube, about the length of the diameter A B, which will turn round C as a pivot. Now fix the protractor horizontally on the top of a staff, and you have a rude kind of theodolite.

3. Suppose it desired to measure distance between P and R, which are separated by a pond, morass, &c. At any convenient point Q plant the staff perfectly upright by means of a spirit level or other contrivance (*vid.* Quadrant); and, having made the tube coincide with A B, turn protractor until the eye at B looking through it sees P. Now keep the *protractor* fixed, but turn the *tube* until the eye looking through it sees R.

The degrees in arc A D tell the size of ∠ P Q R.

Euclid, Book I. *Props.* 4, 15, 26.

1. In questions 83. and 84. on the Triangle, the student is called upon to test his knowledge of the peculiar value of the angles 60°, 90°, and 45°. The diagrams here annexed will illustrate both examples, which show the practical utility of geometrical study.

2. The quadrant or theodolite will be required to determine the angles in both instances.

3. It will often happen, of course, that neither the ∠ of 45° nor 60° can be obtained. When this is the case, another principle, viz. the *equality of opposite vertical ∠s* (Euclid, I. 15.), must be brought into play.

Fig. 68.

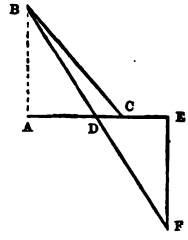
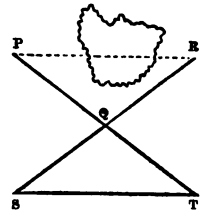


Fig. 69.



4. *e. g.* Suppose it impracticable (in Question 83.) to take an observation at C, where the ∠ is 45°. Take any other distance A D (*fig.* 68.), and having fixed a staff at D, walk forward in the same straight line till the number of paces or yards stepped in D E is the same as those in A D. Plant another rod at E, and having turned your back to the river, walk on in a direction at right ∠s to A E, until at F the 2 points D and B are seen in one line. Then E F is the river's breadth.

For ∠ A D B = ∠ E D F, being opposite vertical ∠s, and ∠ D A B = ∠ D E F, being right ∠s. Also A D = D E. ∴ A B = E F. (Euclid, I. 26.)

5. In question 84. suppose an ∠ of 60° impossible, and P Q, R Q, of any length, equal or unequal. Measure both carefully, and fix a staff at Q (*fig.* 69.) (this being any point from which both P and R can be conveniently seen). Pro-

duce P Q to T, and R Q to S, making Q T=Q P, and Q S=Q R. Now measure S T, and the required distance P R is found.

For ∠ P Q R=∠ S Q T (Euclid, I. 15), and Q R, Q P,= Q P, Q S; ∴ base P R=base S T (Euclid, I. 4.).

6. If ∠ A D B can be determined with quadrant or theodolite, it may be laid down upon paper with the *protractor*, and A D, D E with the *scale of equal parts*. Then at A and E draw two lines ⊥ to A E, and through D draw a line meeting them both. This fixes the position of B and F; and when A B is estimated according to the same scale as A E, its length is found as exactly as though E F had been measured on the ground. This is a considerable saving both of time and labour.

7. When the ∠ P Q R has been observed, and P Q, R Q measured, they may be laid down upon paper in the same way; and therefore S T or P R can be accurately determined from these data without the necessity of actually measuring either Q S, Q T, or S T, with tape or chain.

THE QUADRANT.

1. In the foregoing pages attention has been particularly directed to the importance of the angles 90° and 45°, since by their aid various interesting results may be obtained in the measurement of heights and distances, without encroaching upon the science of Trigonometry (*vid.* Rt. ∠d Δ, *note* 9.).

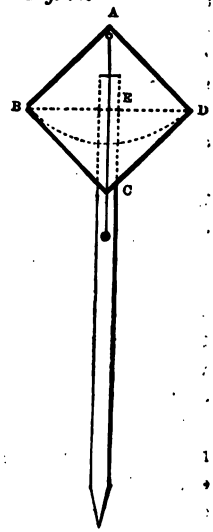
2. The corner of a book, slate, card, &c., will always enable the student readily to lay down a rt. ∠ upon paper; while a *square* card cut in half diagonally will as easily and accurately supply him with an ∠ of 45.° The following simple instrument, which he may construct for himself in a few minutes, will be found of considerable practical utility.

3. Take a square piece of board about 6 inches along each side, and draw the diagonals A C, B D. Then each of the ∠s A B E and B A E = 45°, and A E = B E.

Drill a hole at A, and attach to it a stout thread ten inches long, with a bullet or heavy button tied to the other end, and swinging freely from A as a pivot.

Now fix the square (as in *fig.* 70.) to a rod about six feet long, with a sharp end to enter the ground easily, and the instrument is ready for use.

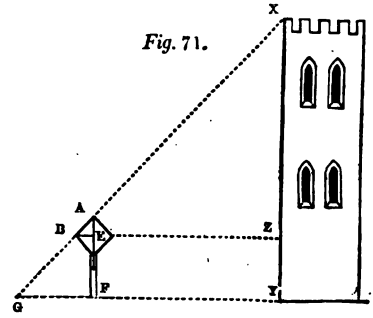
Fig. 70.



4. Since the angle of a square is a rt. ∠, it contains 90°, *i. e.* ¼ of 360°; and the square itself therefore may be called a "quadrant," since each angle contains a *quarter* of the whole number of degrees in a circle.

5. The weighted thread will obviously be an exact test of the quadrant's *uprightness*, since it will not coincide with the diagonal A C if the rod be at all out of the perpendicular.

Fig. 71.



6. Suppose it desired to measure the height of any lofty object X Y, when there is no sun or moon to cast a shadow. First, endeavour to judge as accurately as possible by the eye only, for the sake of practice.

Then plant the quadrant exactly upright at various distances, until at F, the observer, having his eye at B, see

the top of the object, in a line with B A. Now, the large Δ X B Z is exactly similar to the small Δ A B E; and since $\angle$s A B E and B A E each $=45°$, $\therefore$ $\angle$s X B Z and B X Z each $=45°$ too, $\therefore$ X Z $=$ B Z. Measure B Z, *i. e.* F Y, and add to it the length of the rod E F, *i. e.* Z Y. The result is the height of X Y.

7. If this quadrant were fixed horizontally on the top of the rod, and adjusted by means of a small phial of water as a spirit-level, *distances* might be measured on the same principle. (*Vid.* Theodolite.)

8. This square is readily converted into a *sector* (as in *fig.* 70.), whose arc may be divided into 90°. It thus becomes the ordinary quadrant, and can be turned to account in a variety of ways, which the student will find detailed in works of a more advanced character.

9. The student will at this stage hardly need to be reminded that a true square is always readily obtained by describing a circle, drawing two diameters at rt. $\angle$s to each other, and joining their extremities.

MEASUREMENT OF HEIGHTS BY THEIR SHADOW.

It is said that a Greek geometrician travelling in Egypt taught the natives to find the height of the Pyramids by measuring their *shadow*, and comparing it with that cast at the same time by an upright stick of known length. The height of any lofty object whose foot is accessible may thus be readily estimated on a sunny day, in accordance with the theory of similar triangles: *e. g.*, a 6 ft. rod throws a shadow of 8 ft. when a chimney's shadow is 100 ft. Find height of chimney.

Now shadow of rod : shadow of chimney :: rod : chimney,
or, $8 : 100 :: 6 : x$.

$$\therefore 8x = 100 \times 6 \therefore x = \frac{100 \times 6}{8} = 75 \text{ ft.}$$

Examples.

1. Find height of tree whose shadow is 120 ft. at same time that shadow of a 10 ft. pole is 15 ft. *Ans.* 80 ft.

2. Shadow of tower is 140 ft., while a wall 11 ft. high throws one of 16 ft.: find tower. *Ans.* 96¼ ft.

3. A stick 3½ ft. long casts a shadow of 2 ft. 9 in. when a chimney's shadow is 77 ft.: find chimney. *Ans.* 98 ft.

4. A wall measuring ⅜ of a pole in height throws a shadow of 8¼ ft., while shadow of a spire is 32 yds.: find spire. *Ans.* 128 ft.

5. The shadow of a man 5 ft. 10 in. high being 7 ft. 7 in., find height of maypole whose shadow is 32½ yds. *Ans.* 25 yds.

MEASUREMENT OF HEIGHTS OR DEPTHS BY STONE-DROPPING.

1. When the foot or base of a height is not accessible, as in the case of a well, overhanging cliff, &c. &c., an estimate of it may be formed approximately by dropping a stone from the summit, and counting the number of seconds that elapse before it reaches the bottom.

2. For it has been discovered by experiment that any solid body, falling from a state of rest, and not influenced by any force but that of its own gravity, descends about 16 ft. in the first second, 3 times 16 in the 2nd, 5 times 16 in the 3rd, 7 times 16 in the 4th, and so on, its velocity increasing each second as the odd numbers 1, 3, 5, 7, 9, &c.

3. Now if a stone thus falling from a height takes 4 seconds to reach the bottom, the total distance descended will be 16 + 48 + 80 + 112 ft., *i. e.* 256 ft. And this result, though not strictly accurate, is quite near enough to the truth for a rough estimate. [N.B. In default of a watch, 5 beats of the pulse may be reckoned as 4″.]

4. A ready method of performing this calculation is obtained by observing that "every square number is merely the sum of as many of the *odd numbers* (always beginning with 1) as there are units in the number to be squared."

Thus, $2^2 = 1 + 3$; $3^2 = 1 + 3 + 5$; $4^2 = 1 + 3 + 5 + 7$; $5^2 = 1 + 3 + 5 + 7 + 9$; &c. &c. Now in the example just given, the descent was $16.(1 + 3 + 5 + 7)$ or 16×16, which is the same as 16×4^2: and if the fall had occupied 5 seconds, descent would have been 16×25 or (16×5^2) ft., by the same law. Hence, in any case of this kind, the whole operation resolves itself into the simple one of multiplying 16 ft. by the *square* of the number of seconds occupied in the fall.

5. The student, then, will remember this law of descent, —viz. *the distance traversed by a falling body varies as the* SQUARE *of the time spent in falling.*

Examples.

1. From the summit of an overhanging cliff a dropped stone reaches the water in 5″: find height. *Ans.* 400 ft.

2. A stone dropped from a balloon takes half a minute to reach the earth. How high is the balloon? *Ans.* $2\frac{7}{11}$ miles.

3. A stone being dropped into a coalpit, the sound of its fall at the bottom reaches the ear in 6″. Allowing half a second for the transmission of the sound, find depth of pit. *Ans.* 484 ft.

4. A cannon ball fired vertically is 36″ in the air. How high did it ascend? *Ans.* 1728 yds.

MEASUREMENT OF HEIGHTS BY THE BAROMETER.

1. Travellers who have climbed lofty mountains or ascended to great heights in a balloon, all speak of their having on those occasions experienced a painful sensation of fulness or dilatation at the surface of the body, which has in some instances resulted in the rupture of bloodvessels and consequent loss of life.

2. This arises from the *rarefaction of the air, i. e.* its becoming less and less dense or heavy in proportion to its *height* above the ordinary sea level, so that its *downward pressure,* upon which depends its power to control and re-

press the expansive force of the blood, is continually diminishing.

3. Observations of this kind have suggested the use of the barometer in mensuration. For the height of the mercury within the *closed tube* depends upon the weight of air pressing down the mercury in the *open cup;* and consequently the increase or diminution of the weight or pressure of the atmosphere may be estimated exactly by the increase or diminution of mercurial height in the tube.

4. Now it has been found that the mercury falls about $\frac{1}{10}$ inch for every 100 feet of perpendicular ascent; so that if it falls a whole inch in ascending a hill, the height may be estimated approximately as 1000 feet.

Examples.

1. In ascending a hill, the mercury fell $2\frac{1}{2}$ inches: find height. *Ans.* 2500 ft.

2. How much will the mercury fall in ascending a mountain $2\frac{1}{2}$ miles high? *Ans.* $13\frac{1}{2}$ in.

3. In descending this mountain, at a particular point the mercury was found to have risen $2\frac{1}{4}$ in.: find the height of this point above the sea level. *Ans.* 10950 ft.

MEASUREMENT OF DISTANCES BY SOUND AND LIGHT.

1. Sound has been observed to travel in still air at the rate of about 380 yards per second; and as the passage of *light* is almost instantaneous (being 12 million miles per minute, or 200,000 miles per second), distances may be estimated approximately by reference to this standard.

2. Thus, if, after a flash of lightning, 5″ can be counted before the thunder is heard, the storm is rather more than a mile off.

3. Five beats of a healthy pulse may be counted as 4″, since it will beat 75 times in a minute.

Examples.

1. After the flash of a ship's gun at night is seen from a cliff, 17″ are counted before hearing the report: find distance.

Ans. 3 miles, 1180 yds.

2. Between a lightning-flash and thunderclap 14 beats of the pulse are counted: find distance of storm. *Ans.* about $2\frac{1}{2}$ miles.

3. A traveller in the Highlands, using a telescope to watch a deer-stalker, counted 13″ by his watch between the instant of seeing the stag drop and hearing the rifle that killed it: find distance.

Ans. 2 miles, 1420 yds.

4. The great bell of St. Paul's has been heard on a still night as far as Windsor, 17 miles direct. What was the exact London time when the stroke of one o'clock reached Windsor? *Ans.* 1° 1′ $18\frac{14}{19}$″.

5. While standing at the side of a river I found that my voice was re-echoed in 3″ from the opposite bank: find breadth of stream.

Ans. 570 yds.

MEASUREMENT OF THE SPEED GAINED BY INCREASING THE NUMBER OF AGENTS EMPLOYED.

1. In connection with the Cask and Pipe, &c., the following hints will be found useful.

2. When 2 or more agents are employed upon the same piece of work with a view to its more speedy completion, as when 2 pipes are used to fill or empty a cask, &c., and it is required to know how soon it can then be done, the common sense principle of "*finding the value of one*" must be resorted to,—the *one* in this case being the quantity of work done in one minute, hour, day, &c., by all the agents working together.

Ex. 1. Pipe A alone fills a vat in 2°, and B in 3°.

$\therefore$ A fills $\frac{1}{2}$ vat in 1°, and B $\frac{1}{3}$ in 1°.
$\therefore$ $A+B$ fill $\frac{1}{2}+\frac{1}{3}$ (*i. e.* $\frac{5}{6}$) in 1°.
$\therefore$ „ „ $\frac{1}{6}$ vat „ „ $\frac{1}{2}$ hr.
$\therefore$ „ „ $\frac{6}{6}$ or the whole in $\frac{6}{5}$ hr., *i. e.* in 1° 12′.

(*N. B.* 1° 2′ 3″ means 1 hour, 2 minutes, 3 seconds.)

Ex. 2. Pipe *A* fills vat in 2°, whereas *A* and *B* together fill it in 1° 12′. How soon will *B* alone fill it?

Here *A* fills $\frac{1}{2}$ vat in 1°, and $A+B$ fill $\frac{5}{6}$ in 1° (as before).

$\therefore$ *B* fills $(\frac{5}{6}-\frac{1}{2})$ *i. e.* $\frac{1}{3}$, in 1°.

$\therefore$ „ $\frac{3}{3}$, or whole in 3°.

Ex. 3. *A* fills in 2°, and *B* in 3°, while *C* would *empty* it in 4°. How long will it take to fill when all 3 are open together?

In this case, work done in $1^\circ = \frac{1}{2}+\frac{1}{3}-\frac{1}{4}$, &c.

Ex. 4. *A* fills in 4° and *B* in 6°, while *A* empties in 2°. Here the work done in $1 = \frac{1}{4}+\frac{1}{6}-\frac{1}{2} = -\frac{1}{12}$, *i. e.* the fluid pours *out* faster than it pours *in*, and consequently the vat will never fill at all.

Ex. 5. Supposing these 3 pipes to commence running when the vat is *full*, how soon will it be empty?

Since $\frac{1}{12}$ is emptied in 1°, $\therefore \frac{12}{12}$, or the whole, will be emptied in 12°.

Ex. 6. *A* fills in 2° and *B* in 3°. How soon will it be full if *A* is open 15′ before *B* begins running?

In this case *A* fills $\frac{1}{4}$ vat before *B* begins: $\therefore$ only $\frac{3}{4}$ remain to be filled by their joint action. Now they together fill the whole vat in 72′ (*vid. Ex.* 1.) $\therefore$ they will fill $\frac{3}{4}$ of it in $\frac{3}{4}$ of 72′, *i. e.* 1° 3′. And consequently the whole time of filling is 63′ + 15′, or 78′; *i. e.* 6′ longer than if they had started together.

Examples.

1. Pipe *A* fills a vat in 3°, *B* in 4°, *C* in 6°. How soon will it be filled if all run at once? *Ans.* 1° 20′.

2. *A* takes 6° to fill a vat, which *A* and *B* together fill in 4°. Find *B's* time of doing it alone? *Ans.* 12°.

3. *A* fills a cask in $\frac{7}{11}$°, and *B* in $\frac{1}{8}$°. Find time of filling it jointly. *Ans.* 5′ 15″.

4. *A* and *B* together do a piece of work in $2\frac{1}{7}$ days, which *A* alone can do in $4\frac{1}{2}$ days. Find *B's* time of doing it alone. *Ans.* 6 days.

5. Pipe *A* fills a barrel in 6′ and *B* in 3′, while *C* will *empty* it in 4′. How soon before full, when all open together? *Ans.* 4′.

6. Tap *A* fills a cistern in 15′, and *B* in 30′, which *C* can empty in 10′. How long before full? *Ans.* Never.

7. If *full*, how long before empty? *Ans.* Never.

8. *A* fills in 40′, *B* in 30′, *C* in 50′, while *D* and *E* can each empty in 25′. If full, how long before empty? *Ans.* 10′.

9. *A*, *B*, *C*, fill tank in $1\frac{1}{2}$°, $2\frac{1}{4}$°, $2\frac{1}{2}$°; while *D*, *E*, *F* empty in $1\frac{1}{4}$°, $2\frac{3}{4}$°, $3\frac{3}{4}$°, respectively. How long before full? *Ans.* 12° 22′ 30″.

10. *A*, *B*, *C*, together fill vat in 2°; *A* and *B* in 3°; *A* and *C* in 4°. How long will each be filling it alone? *Ans.* *A* 12°, *B* 4°, *C* 6°.

11. How soon will vat in Ex. 1. be filled, if B is open for an hour before the other two? *Ans.* 1°.

12. A, B, C, do work in 12, 10, 8 days. Find time of doing it when C begins $\frac{1}{2}$ day before A and B. *Ans.* $3\frac{39}{77}$ days.

13. A, B, C jointly fill in 3°; A and B in 3° 54′; A and C in 4° 20′. Find time of each doing it alone. *Ans.* A $6\frac{1}{2}$°, B $9\frac{3}{4}$°, C 13°.

14. A, B, C fill in 15′, 20′, 35′, while D and E empty in 45′ and 10′. How soon will it be full if A and D are open 3′ before the rest? *Ans.* $37\frac{11}{35}$°.

15. Through A, B, C, filling pipes of a vat which holds 6900 gals., 200 gals. each flow in 15′, 16′, 18′; while D and E each discharge 200 gals. from it in 12′ and $7\frac{1}{2}$′. If full, how soon empty? *Ans.* 18°.

MEASUREMENT OF TIME AND DISTANCE, AS CONNECTED WITH RATE OF SPEED.

1. A and B start at same time to meet each other from X and Y, two places 21 miles apart. A walks 3, and B 4 miles an hour. How soon and where will they meet?

Here again, the grand point is to "find the value of *one*," *i. e.* either the time spent in the joint performance of *one mile*, or the joint distance done in *one hour*; for since A and B are *meeting* each other, the whole distance is traversed by neither of them singly, but by the combined exertions of both.

[This principle has just been illustrated in the case of two or more pipes helping to fill the same cask.]

Now 3 + 4 miles are done by A and B in 1°.

$\therefore$ 1 „ „ $\frac{1}{7}$°.

$\therefore$ 21 „ „ $\frac{21}{7}$° or 3°,

at which time of meeting they will be 9 miles from X and 12 from Y, since A has travelled 3 times 3 miles, and B 3 times 4 miles.

2. If A starts an hour before B, then the question is as though they started together from a distance of 18 miles

asunder; and the result is obtained in the same way as before.

3. *A* walking 3 miles an hour leaves X at 8 A.M.; *B* follows him at rate of 4 miles an hour, starting at 9 A.M. How soon will *A* be overtaken?

[*N. B.*—Questions of this kind may of course be done by Algebra; but the following consideration will render its application needless, by indicating a clear and simple method of solving the problem without it.]

A has 3 miles start. To overtake him, therefore, *B* must walk faster, *i. e.* must get over not only all the ground that *A* covers, but a *little more*, within the hour; and he must continue to walk on, until the sum of these *little mores* shall have equalled the total distance that *A* was ahead when *B* started. The instant this is done, *B* reaches the object of his pursuit. The one purpose, then, of *B*'s *excess of speed* over *A*, is the *diminishing of this distance ahead:* and accordingly, by comparing the two, the desired result is at once obtained.

Thus: — Between 9 and 10, *A* walks 3 miles, and *B* (3 + 1) miles. And the same thing occurs in each of the 2 hours following; at which time *A* will have travelled 4 times 3 miles, and *B*, 3 times (3 + 1) miles; *i. e.* *A* is overtaken in 3 hours, because *B*'s *excess of speed* per hour is $\frac{1}{3}$ of *A*'s distance ahead.

4. Had *B* walked $4\frac{1}{2}$ miles per hour, he would have caught *A* in 2 hours instead of 3, since his $1\frac{1}{2}$ mile per hour greater speed is just $\frac{1}{2}$ of *A*'s 3 mile start.

Examples.

1. A vessel sailing 9 miles an hour is 18 miles ahead of a steamer which goes 12 miles an hour. How soon will she be overtaken?

Ans. 6 hrs.

[*N. B.*—The steamer's extra speed of 3 miles an hour is just $\frac{1}{3}$ of the 18 miles start of the sailer; ∴ in 2′ she will have gained $\frac{2}{3}$; in 3°, $\frac{3}{3}$, &c. &c.; and in 6°, $\frac{6}{3}$ or the whole 18 miles.]

2. *A* has 15 miles start of *B* who travels 10 miles an hour to *A*'s 8. How soon is *A* caught? *Ans.* $7\frac{1}{2}$ hrs.

3. *A* travelling 7, and *B* 6, miles an hour, start at same time from *M* and *N*, which are 65 miles asunder. When and where will they meet? *Ans.* 5 hrs., 35 miles from *M*.

4. In a 2 mile race, *A* has 109 yds. of *B*, whose stride is $72\frac{1}{4}$ inches to *A's* 72. Which will win, and by how much? *Ans.* *B* 1 yd.

5. *A* walks $4\frac{1}{2}$ miles per hour and *B* $3\frac{3}{4}$ miles. How much sooner will *A* walk 25 miles than *B*? *Ans.* 1° 6′ 40″.

6. *A* walks $1\frac{1}{2}$ mile in $22\frac{1}{2}'$ and *B* $2\frac{1}{2}$ miles in half an hour. How soon will they meet if they set out at same time from 2 places $13\frac{1}{2}$ miles apart? *Ans.* 1° 30′.

7. *A* trots 7 miles in 45′, *B* 7 miles in 50′. When will they meet if they start at same time from 2 places $31\frac{1}{4}$ miles asunder? *Ans.* 1° 52′ 30″.

8. A 20 miles an hour train leaves London at 7 A. M., and a 25 do., do. at 9. How far behind will 2nd train be when 1st reaches Chester, 200 miles off? *Ans.* They arrive together.

9. A merchant ship is $9\frac{3}{8}$ miles ahead of a pirate who sails 8 miles an hour, and overtakes her after a 50 miles chase. Find trader's speed. *Ans.* $6\frac{1}{2}$ miles per hr.

10. *A*, 30 miles an hour train, leaves London at 7 A. M., and express *B*, 45 miles an hour, starts at 9. Where and when must *A* be shunted off the main line to let *B* pass? *Ans.* 1 P. M., 180 miles.

11. In a mile race, *A* has 80 yds. start of *B*, who gains 5 ft. in every 35 yds. Which will win, and by how much? *Ans.* *B* $3\frac{17}{21}$ yds.

12. *A* walks $3\frac{1}{3}$ miles an hour, and *B* $4\frac{1}{6}$ miles. How long will it take *B* to catch up *A* who is $6\frac{2}{3}$ miles ahead? *Ans.* 6° 24′.

13. One horse trots 9 miles in 32′, and another $7\frac{1}{2}$ in 28′. Which is the swifter, and how much start must the slower have in a match of 21 miles, so that they may run a dead heat? *Ans.* 1 mile.

14. *A* and *B* start at same time to a place *C*, 150 miles off. *A* travels 3 miles an hour faster than *B*, and reaches *C* 8° 20′ before him. Find rate of each. *Ans.* 9 and 6 miles.

15. A thief, walking $4\frac{1}{2}$ miles an hour, has 2° 30′ start of a constable who drives at rate of $8\frac{1}{4}$ miles per hour. How far must *C* go before he can effect a capture? *Ans.* $24\frac{3}{4}$ miles.

16. In a race of 1700 yds. *A* has 100 yds. start of *B*, who runs $12\frac{3}{4}$ inches to *A's* 12. Which will win? *Ans.* A dead heat.

17. A 20 miles an hour train starts at $8\frac{1}{2}$ A. M. and a 25 miles an hour follows at 10. Where and when must the former be shunted on to a siding to let the latter pass? *Ans.* 2° 30′, 120 miles.

18. Two towns are 324 miles apart. *A* travels 51 miles, when *B* follows him at rate of 19 miles per hour to *A's* 16. Which will do the distance first? *Ans.* *B*. $35\frac{19}{19}$′.

19. Two persons start at same time from opposite points $52\frac{1}{2}$ miles apart. *A* travels half as fast again as *B*, who walks $\frac{1}{15}$ of the whole distance in an hour. Where and when will they meet?

Ans. 6°, *A* $31\frac{1}{2}$ miles.

20. A thief, driving a cart $7\frac{1}{2}$ miles per hour, has $1\frac{1}{2}$° start of a constable, who follows him in a gig at rate of 9 miles an hour, and starts at 7° 30′ A. M. Pace being uniform and chase continuous, find time of capture and distance travelled. *Ans.* 3 P. M., $67\frac{1}{2}$ miles.

21. A hare starts up 75 yds. before a dog, but runs only $5\frac{1}{3}$ ft. to dog's $6\frac{3}{4}$ ft. How soon will she be caught?

Ans. 405 yds. from dog's start.

22. A steam cruiser, going 8 miles an hour, descried a slaver $5\frac{1}{2}$ miles ahead, going 7 miles an hour, and distant only 21 miles from her destination. Accordingly the steamer increased her speed, and managed to overhaul the slaver when just within reach of her port? What was the increase? *Ans.* 1 mile.

23. *A* and *B* are 270 miles apart. A 22 miles an hour train leaves *A* at 10° 30′ A. M., and a 28 miles an hour leaves *B* at 1° 15′ P. M. Where and when will they meet?

Ans. $\begin{cases} 5° 26' 24'' \text{ P. M.} \\ A\ 152\frac{17}{45},\ B\ 117\frac{8}{45} \text{ miles.} \end{cases}$

24. A thief has 18 miles start of a constable, who travels $9\frac{1}{2}$ miles an hour, and secures his prisoner at 4° 30′ P. M., having started at 8° 30′ A. M. Find thief's pace and distance travelled.

Ans. $7\frac{1}{4}$ and 58 miles.

25. A smuggler has $7\frac{3}{4}$ miles start of a revenue-cutter, which sails 6 ft. to the smuggler's 5, and runs 9 miles an hour. How long will the chase last? *Ans.* 5° 10′.

26. Three horses enter for a handicap.

A's speed is known to be to *B's* as 5 : 4.
B's „ „ *C's* as 5 : 4.

A being weighted at 10 stone 10 lbs., what must be the weights of *B* and *C* to put them on an exact equality?

Ans. 8 st. 8 lbs. and 6 st. 12 lbs.

MEASUREMENT OF SPEED IN A STEAM VESSEL, AS CONNECTED WITH THE CONSUMPTION OF COAL; AND VICE VERSA.

1. On a first glance at the subject, it might be imagined, reasonably enough, that if any given quantity of coal drive a steamer through the water at a particular rate, the consumption of twice the quantity in the same time would drive her at twice the speed; and so on. But this is far from being the case in reality, and for reasons like the following.

2. A boat which moves 5 miles an hour displaces certain particles of water with a certain velocity. If it moves twice as fast, it displaces *twice as many* particles with *twice the velocity;* if thrice as fast, *thrice* as many with *thrice the velocity*, &c. &c. So that in the former case 4 times, and in the latter 9 times, as much coal or steam power are required as at first. (Arnott's "Physics.")

3. Thus, if a consumption of 4 tons daily produce power enough to drive a ship 5 miles an hour, the coal requisite for (5×2) or 10 miles an hour will be (4×2^2) or 16 tons; for (5×3) or 15 miles (4×3^2) or 36 tons. And so on.

4. In the same way it will be seen that if speed be *doubled*, the fuel will last only a *fourth* of the time or distance; if *trebled*, a *ninth*, &c. &c. While if the distance to be done is *quadrupled*, speed must be *halved;* if distance becomes 9 times as great, speed must be reduced to $\frac{1}{3}$, &c. &c.

5. And this is a highly important consideration; since a steam ship may be blown out of her course or meet with accidents, like any other; and in such cases it becomes essential to know what reduction must be made in speed in order that the fuel may last out the required distance.

6. *E. G.* Suppose a steamer, with coals enough for 200 miles at 6 knots an hour, to meet with an accident requiring immediate repair. Then her fuel will just serve to carry her at double speed to a port 50 miles off. Whereas if she should be blown off by a storm to a distance of 800 miles,

the same fuel may be made to bring her home, by reducing speed to 3 knots an hour.

7. A solid ton of coal occupies about a cubic yard or 27 ft. When broken it will occupy about 40 cubic ft. And the dimensions of a ship's hold being known, the quantity of fuel in it at any particular moment can be readily discovered by simple measurement.

8. Theory, then, would make the consumption of coal vary as the *square* of the speed. Practice, however, shows that this estimate is considerably below the mark, and that it is not safe to act on any other rule than that "consumption of fuel varies as *cube* of speed;" *i. e.* that 2^3 or 8 tons will be required to give *twice* the speed of 1 ton; 3^3 or 27 tons to give thrice the speed; and so on.

Ex. 1. A ship going 5 knots an hour and using $4\frac{1}{2}$ tons a day, increases her speed to 10 knots. What will be her new rate of coal consumption?

Here, $5^3 : 10^3 :: 4\frac{1}{2} : x.$

And since "product of extremes = product of means,"

$$\therefore\ x \times 5^3 = 4\tfrac{1}{2} \times 10^3 \quad \therefore\ x = 36 \text{ tons.}$$

Or thus, 10 is 2 times 5; $\therefore$ coal consumed must $= 2^3$ times $4\frac{1}{2} =$ 36 tons.

Ex. 2. A steamer using $13\frac{1}{2}$ tons daily with 9 knots speed, increases to 12 knots. Find coal consumption.

Here, $9^3 : 12^3 :: 13\frac{1}{2} : x.$ $\therefore$ (as before) $x = 32$ tons.

Or thus, 12 is $\frac{4}{3}$ of 9, $\therefore$ coal $= \frac{4^3}{3^3}$ of $13\frac{1}{2} = \frac{64}{27}$ of $13\frac{1}{2} = 32$ tons.

Ex. 3. After chasing for some time at 12 knots speed, with an expenditure of $1\frac{1}{4}$ ton per hour, the ship slackened to 8 knots. Find diminished daily consumption.

Here, $12^3 : 8^3 :: 36 \text{ tons} : x.$ $\therefore$ (as before) $x = 10\frac{2}{3}$ tons.

Or thus, $8 = \frac{2}{3}$ of 12, $\therefore$ coal $= \frac{2^3}{3^3}$ of $36 = \frac{8}{27}$ of $36 = 10\frac{2}{3}$.

Ex. 4. At 8 knots speed a ship had been using 16 tons of coal daily, until she found herself still 432 miles from home with only 144 tons. How much must she reduce speed to make them last?

At 8 knots, she is 54 hours from port, which would require 288 tons, or twice the quantity she has. Then,

$$288 \text{ tons} : 144 :: 8^3 \text{ knots} : x^3 ; \ \therefore x = 6\tfrac{1}{3} \text{ knots.}$$

Or thus; coal being $\frac{1}{2}$, speed must be $8 \times \frac{1}{\sqrt[3]{2}} = 6\frac{1}{3}$.

[For if 8 times as much coal were used, speed would be 8×2, *i. e.* $8 \times \sqrt[3]{8}$. Note 8.]

Examples

1. A steamer having used 6 tons of coal daily at speed of $5\frac{1}{2}$ knots an hour, suddenly increased it to 11 knots. Find new daily consumption. *Ans.* 30 tons.

2. A steamer having used 10 tons with 7 knots, increased to $10\frac{1}{2}$ knots. Find her daily consumption. *Ans.* $33\frac{3}{4}$ tons.

3. A steamer having used 20 tons with 8 knots, increased to 10 knots. Find her daily consumption. *Ans.* $39\frac{1}{16}$ tons.

4. A steamer having used 40 tons with 12 knots, reduced to 20 tons daily. Find diminished speed. *Ans.* $9\frac{1}{2}$ knots.

5. A steamer having used 30 tons with $10\frac{1}{2}$ knots, reduced speed to $5\frac{1}{4}$ knots. Find reduced consumption. *Ans.* 6 tons.

6. A steamer having used 33·75 tons with 11 knots, reduced to $7\frac{1}{3}$ knots. Find reduced consumption. *Ans.* 10 tons.

7. Between Malta and Gibraltar (1100 miles) a steamer consumed 250 tons, with a speed of 11 knots; but between Gibraltar and Portsmouth (1100) reduced speed to 8 knots. Find consumption of coal during this latter portion of the voyage. *Ans.* $132\frac{1}{2}$ tons.

8. On the return voyage this vessel took out 400 tons of coal, and used 260 tons, with a speed of 10 knots, from England to Gibraltar. What must be her speed to Malta to make the coals last out?

Ans. 7·3 knots.

9. A ship carrying 300 tons of fuel kept up a speed of 11 knots for 5 days, with consumption of 36 tons daily. Her port was then found to be distant 1660 miles. At what speed can her coals be made to hold out? *Ans.* 8 knots.

10. At 8 knots speed, a steamer made 1000 miles with 80 tons of coal. How much would she need for 1500 miles at 10 knots speed?

Ans. 187 tons.

MEASUREMENT OF WATER-PRESSURE.

1. Since a cubic foot of water weighs 1000 oz. or 62½ lbs., and since the mass and weight of any body of water increases with its depth, it becomes important to know what is the proportion of this increase, so that due strength may be given to the sides of a wall, embankment of earth, &c., to enable it to resist the pressure of any given volume, depth, &c. of fluid.

2. Careful experiments have established that this increase is in arithmetical progression, being in the ratio of 1, 3, 5, 7, 9, &c.: so that whatever be the pressure at a depth of 1 ft., at 2 ft. the pressure is three times; at 3 ft. 5 times, at 4 ft., 7 times as great, &c. &c. The resisting power of the embankment, therefore, must be in the same proportion, or the consequence will be an inundation.

3. After having taken several examples of this kind, the student will observe a *law*, according to which the resisting power may at once be estimated for any given depth of water; since he will notice that if x ft. be the depth of water, the strength of wall at that depth is $(2x-1)$ times what it is at depth of 1 ft.

4. Thus if a reservoir be 6 ft. deep, the thickness or strength of the embankment at the bottom must be $(12-1)$ times greater than at the top; if 7 ft. $(14-1)$ times; if 8 ft. $(16-1)$ times, &c. &c.

5. When it is required to know what the total weight or pressure of water will be upon the sides of a tank, locks of a canal, dock-gates, &c., with a view to their proper construction, it may be determined by finding the extent of surface exposed to the water's action, and multiplying this area by the water's *mean depth*.

e. g. A tank is (13 × 10) ft. and 6 ft. deep.
Here, area of one *side* is (13 × 6) ft. or 78 sq. ft.
„ *end* is (10 × 6) ft. or 60 do.

And mean depth is $\frac{1}{2}$ of 6 ft., *i. e.* 3 ft.
$\therefore$ cubic feet of water pressing each *side* $= (78 \times 3)$, or 234.
and " " *end* $= (60 \times 3)$, or 180.
And since 1 cub. ft. water weighs 1000 oz. or $62\frac{1}{2}$ lbs.,
$\therefore$ total weight or pressure on each *side* $= 62\frac{1}{2} \times 234 = 14625$ lbs,
and " " *end* $= 62\frac{1}{2} \times 180 = 11250$ lbs.

6. Seawater, being impregnated with salt, &c., is heavier than fresh; and it is therefore usual to allow 64 lbs. to the cubic foot.

7. The reason for taking *mean depth* will be seen by writing down the proportional pressures at each successive foot's depth in the last example, viz. 1, 3, 5, 7, 9, 11. Here observe that the smaller pressure near the surface exactly compensates for the greater pressure lower down; so that the two pressures at the first and sixth ft. together = the sum of the 2nd and 5th; and also = the sum of the 3rd and 4th,—being 12 in each case.

Examples.

1. Two reservoirs are 7 and 10 ft. deep. How much stronger must each be at bottom than at top? *Ans.* 13 and 19 times.

2. The depth of water in a dock is to be increased from 17 to $22\frac{1}{2}$ ft. by constructing an embankment at the top. How much stronger must the new gates be made at bottom than before? *Ans.* $\frac{1}{2}$.

3. These dock-gates being 15 yds. wide, find water-pressure upon them in each case. *Ans.* { 185 tons, 15 cwt. 2 qrs. 24 lbs. / $325\frac{25}{28}$ tons.

4. A canal lock is 15 ft. wide, and its two gates are 7 and 21 ft. deep respectively: find pressure upon each when full.

Ans. { 10 tons, $5\frac{5}{84}$ cwts. / 92 tons, $5\frac{45}{84}$ cwts.

5. A cistern is (4×3) ft. and $2\frac{1}{2}$ ft. deep: find pressure on each side and end when full. *Ans.* $781\frac{1}{4}$ and $585\frac{15}{16}$ lbs.

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3. *A* travelling 7, and *B* 6, miles an hour, start at same time from *M* and *N*, which are 65 miles asunder. When and where will they meet? *Ans.* 5 hrs., 35 miles from *M*.

4. In a 2 mile race, *A* has 109 yds. of *B*, whose stride is $72\frac{1}{4}$ inches to *A's* 72. Which will win, and by how much? *Ans.* *B* 1 yd.

5. *A* walks $4\frac{1}{2}$ miles per hour and *B* $3\frac{3}{4}$ miles. How much sooner will *A* walk 25 miles than *B*? *Ans.* 1° 6′ 40″.

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Ans. $6\frac{1}{2}$ miles per hr.

10. *A*, 30 miles an hour train, leaves London at 7 A. M., and express *B*, 45 miles an hour, starts at 9. Where and when must *A* be shunted off the main line to let *B* pass? *Ans.* 1 P. M., 180 miles.

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12. *A* walks $3\frac{1}{8}$ miles an hour, and *B* $4\frac{1}{8}$ miles. How long will it take *B* to catch up *A* who is $6\frac{3}{4}$ miles ahead? *Ans.* 6° 24′.

13. One horse trots 9 miles in 32′, and another $7\frac{1}{2}$ in 28′. Which is the swifter, and how much start must the slower have in a match of 21 miles, so that they may run a dead heat? *Ans.* 1 mile.

14. *A* and *B* start at same time to a place *C*, 150 miles off. *A* travels 3 miles an hour faster than *B*, and reaches *C* 8° 20′ before him. Find rate of each. *Ans.* 9 and 6 miles.

15. A thief, walking $4\frac{1}{2}$ miles an hour, has 2° 30′ start of a constable who drives at rate of $8\frac{1}{4}$ miles per hour. How far must *C* go before he can effect a capture? *Ans.* $24\frac{3}{4}$ miles.

16. In a race of 1700 yds. *A* has 100 yds. start of *B*, who runs $12\frac{3}{4}$ inches to *A's* 12. Which will win? *Ans.* A dead heat.

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